Probabilistic Smart Contracts:

Secure Randomness on the Blockchain

Krishnendu Chatterjee*, ***Amir Goharshady****, Arash Pourdamghani**

*IST Austria, **Sharif University of Technology

Random Numbers on the Blockchain

- *Current programmable blockchains do not allow probabilistic behavior.*
- *Probabilistic programs are much more general than non-probabilistic programs.*
- *Many financial contracts (e.g. lotteries and gambling) are inherently probabilistic.*
- *Random number generation can be used for proof-of-stake mining.*
- *Many distributed algorithms and protocols rely on randomness.*

All names, characters, businesses, places, events and incidents portrayed in this talk are either the products of the author's imagination or used in a fictitious manner. Any resemblance to actual persons, living or dead, or actual events is purely coincidental. I am a poor graduate student who cannot afford legal fees.

Do not sue me,

The Lottery Story

- *Ed, a well-known celebrity and billionaire, is rolling the raffle drum 4 times to find a winner.*
- *When the number 8 comes out in the 2nd draw, Ed says he hates this number, puts it back in the drum, and rolls it again.*
- *Has Ed cheated?*

The Lottery Story

- *Turns out Ed had bought half of all the tickets.*
- *He did not buy any tickets with 8 in them.*
- *By this trick, he increased his chance of winning the lottery.*

No-redraw rule:

Redrawing is cheating!

Ed should not be able to change the results.

The Lottery Story

- *Next year, the organizers ban redraws.*
- *Ed is rolling the drum again.*
- *The number 8 never appears in the rolls.*
- *Turns out Ed has bribed the drum maker.*

<u>No-centralization rule:</u>
Centralization is cheating!
No central authority (including the lottery organizers) should make or roll the drums.

The Lottery Story

- *Next year, the organizers invite 4 celebrities.*
- *They each bring their own drum.*
- *Each celebrity draws a number and announces it. Ed is last.*
- *Ed wins again!*

<u>Concurrency rule:</u>
Everyone should draw at the same time!
(or at least before knowing other draws)

The Lottery Story

- *Next year, the organizers enforce concurrency.*
- *Ed does not announce his number.*
- *He just walks away.*
- *The organizers have to invite another celebrity for the 4^{th} draw.*
- *Ed wins.*

<u>Penalty rule:</u>

There should be a penalty for not announcing the draw.

The penalty should be at least as big as the lottery prize itself.




The Lottery Story

- *Next year, the organizers enforce penalties using deposits.*
- *Ed wins.*

<u>Rule of 1:</u>

Even if one participant is generating uniformly random draws, the whole result should be uniformly random.



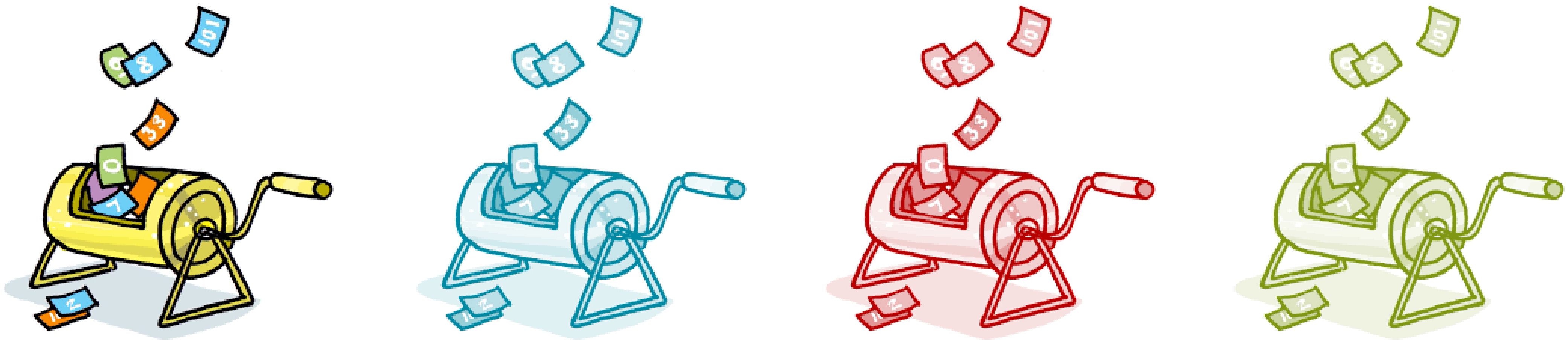

The Lottery Story

- *Next year, each participant draws 4 times.*
- *The result is determined by XORing.*
- *Ed wins.*
- *Turns out he bribed every participant.*

<u>*Openness:*</u>

Drawing should be open to everyone.

Let's do it on the blockchain!



The Lottery Story

- *Next year, anyone who can pay the deposit can participate.*
- *The result is determined by XORing.*
- *Ed wins.*
- *Turns out no one is willing to participate and deposit money without being paid.*

<u>Incentivization:</u>
Each participant should be paid for their input.




The Lottery Story

- *Next year, anyone who can pay the deposit can participate. Each participate receives a reward for providing random numbers.*
- *Ed wins.*
- *Turns out participants did not buy drums. They just reported 0s as the result.*

Incentivization:

Each participant should be paid for their input. It should also be in their best interest to provide uniformly random inputs.




More on Incentives

- *Consider a classic one-shot game with n players.*
- Nash Equilibrium: No player is willing to change strategies.

What if the players can collude?

- Strong Nash Equilibrium: No set of players can collude to change strategies so that all of them profit.

What if the players can share rewards?

- Quasi-strong Nash Equilibrium: No set of players can collude to change strategies so that their *total payoff* increases.

Previous Approaches

- *Relying on block hash/timestamp* (Ed is the miner)
- *Using an oracle* (Ed is the oracle owner)
- *Using commitment schemes* (No incentivization for random inputs)
  - *In the registration phase:*
    - *Each participant pays a deposit*
    - *They commit to a bit b, by submitting hash(b, nonce, id).*
  - *In the revealing phase:*
    - *Each participant reveals their nonce*

The generated random bit is the XOR of all submitted bits.

Rewards for each participant who reveals the correct nonce. Confiscation of deposit for others.

Our Approach

- Use commitment schemes
  - but *let the reward depend on the submitted random bits*

- Make it a game where submitting *uniformly random* bits is the only quasi-strong equilibrium

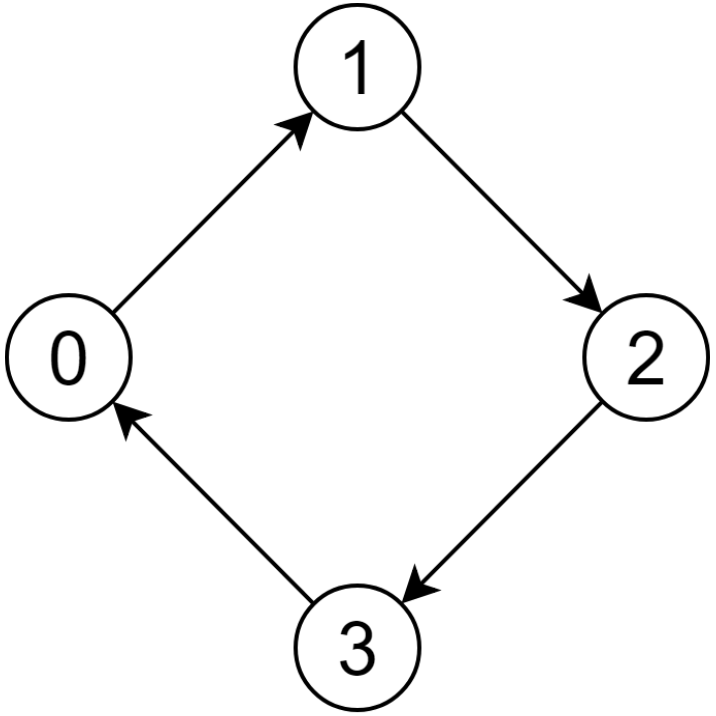
The Game

- n players.
- An even-numbered player can play either 0 or 2.
- An odd-numbered player can play either 1 or 3.
- Let's say that player i plays s_i. Then the utility for player i is:

$$u_i(s_1, \ldots, s_n) := \Sigma_{j \neq i} \; f(s_i, s_j)$$

where

$$f(s_i, s_j) := \begin{cases} 1 & s_i \equiv s_j + 1 \pmod{4} \\ -1 & s_i \equiv s_j - 1 \pmod{4} \\ 0 & \text{otherwise} \end{cases} .$$



The Overall Protocol

- Implemented as a solidity smart contract that can be called by other contracts for generating random bits.
- Consists of 6 steps:

1. Another contract/node requests a random bit and sets the penalty and the reward.
2. Participants can register in a given timeframe. To register, they should provide:
   - A deposit equal to the penalty
   - hash(b, nonce, id)
3. In a given timeframe after the registration, each participant has to reveal their nonce.
4. The deposits are paid back.
5. The game is played and the rewards are calculated. $r_p := \alpha \cdot \left(1 + u_p(s) / n'\right)$
6. The output is the xor of the submitted bits.

Guarantees

Secure Randomness on the Blockchain

- No-redraw rule (by design)
- No-centralization rule (by design)
- Concurrency rule (commitment schemes)
- Penalty rule (by design)
- Rule of 1 (due to XOR)
- Openness (anyone can register)
- Incentivization (due to the game)
- Safety against malicious miners (block withholding, DoS)