SeedTree: A Dynamically Optimal And Local Self-Adjusting Tree

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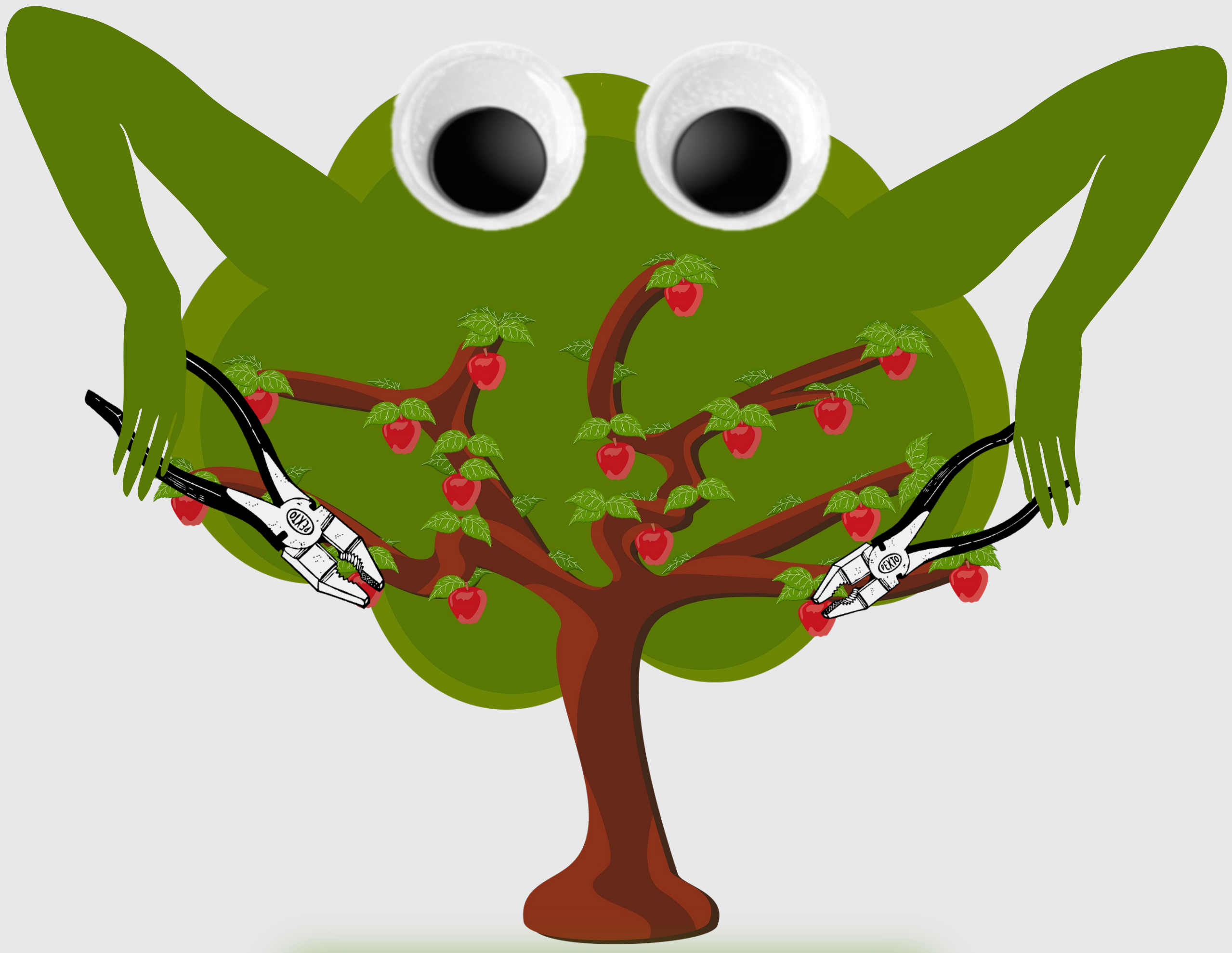
Joint work with Chen Avin, Robert Sama, Stefan Schmid

Dutch Optimization Seminar

29 June 2023

An adopted version of INFOCOM’23 Talk



SeedTree at a glance

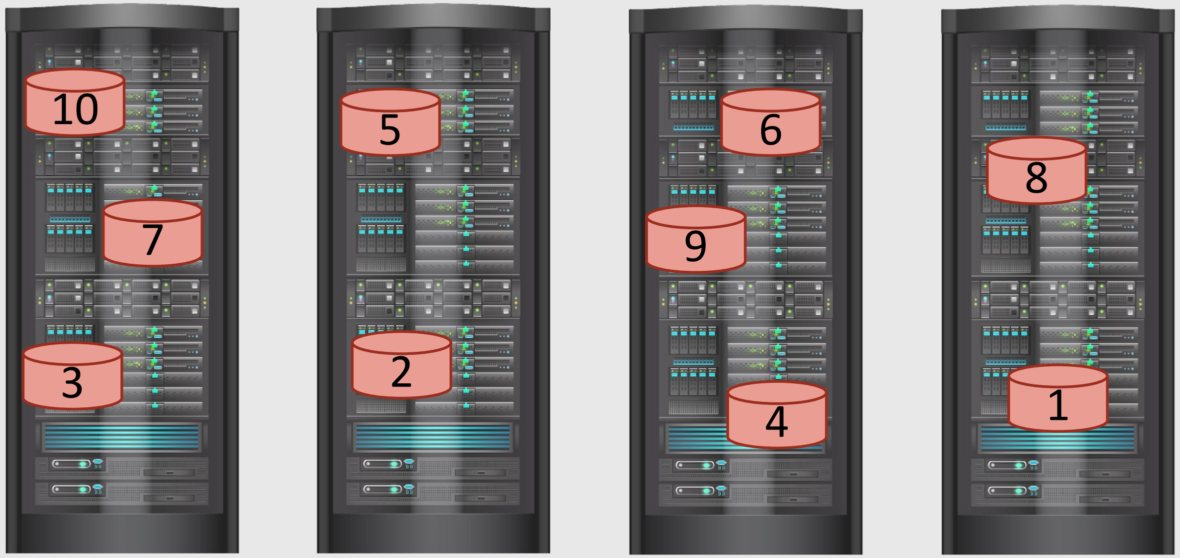
SeedTree

1) Why? 2) What? 3) How?

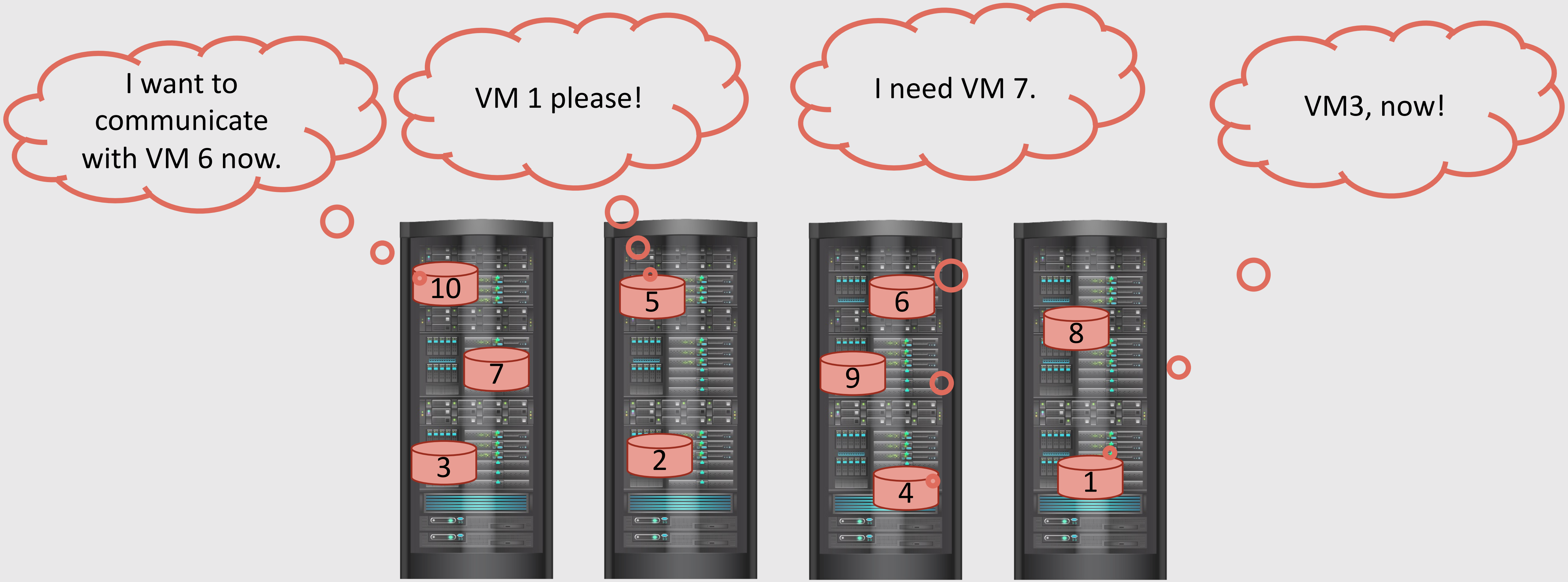
SeedTree

1) Why? 2) What? 3) How?

Servers and VMs



Online Request Sequence



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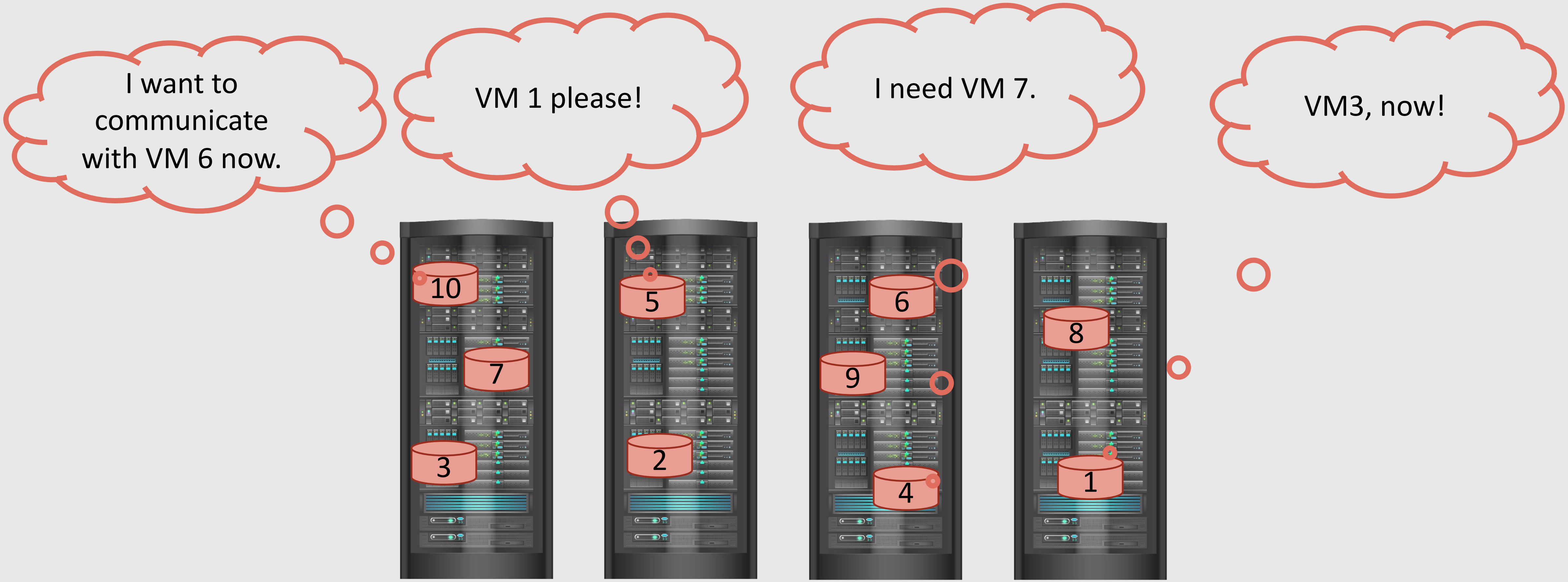
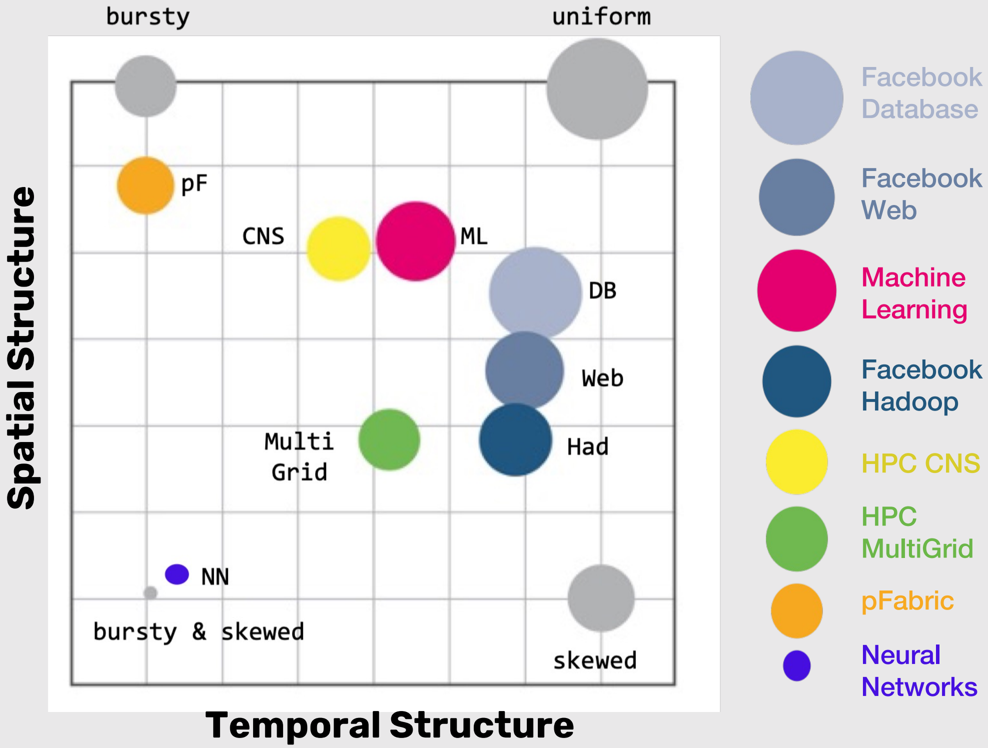


Do we have any structure in the demand?

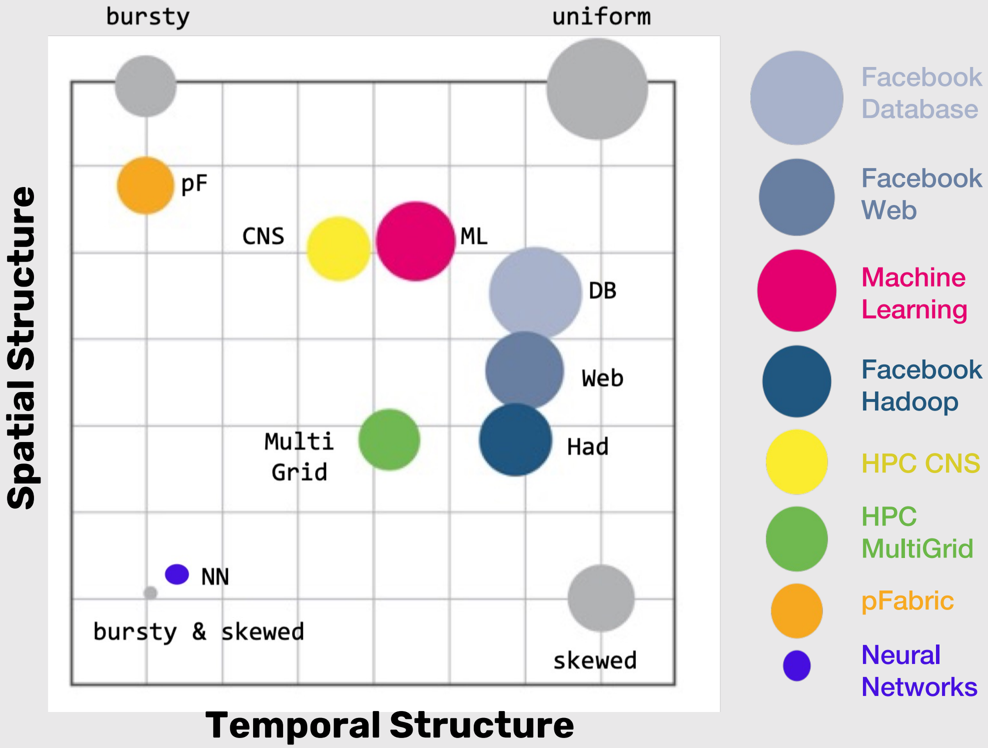
Structure in The Demand



On the Complexity of Traffic Traces and Implications
Avin et al., ACM SIGMETRICS 2020

Structure in The Demand

Can we design a self-adjusting network that utilizes demand?



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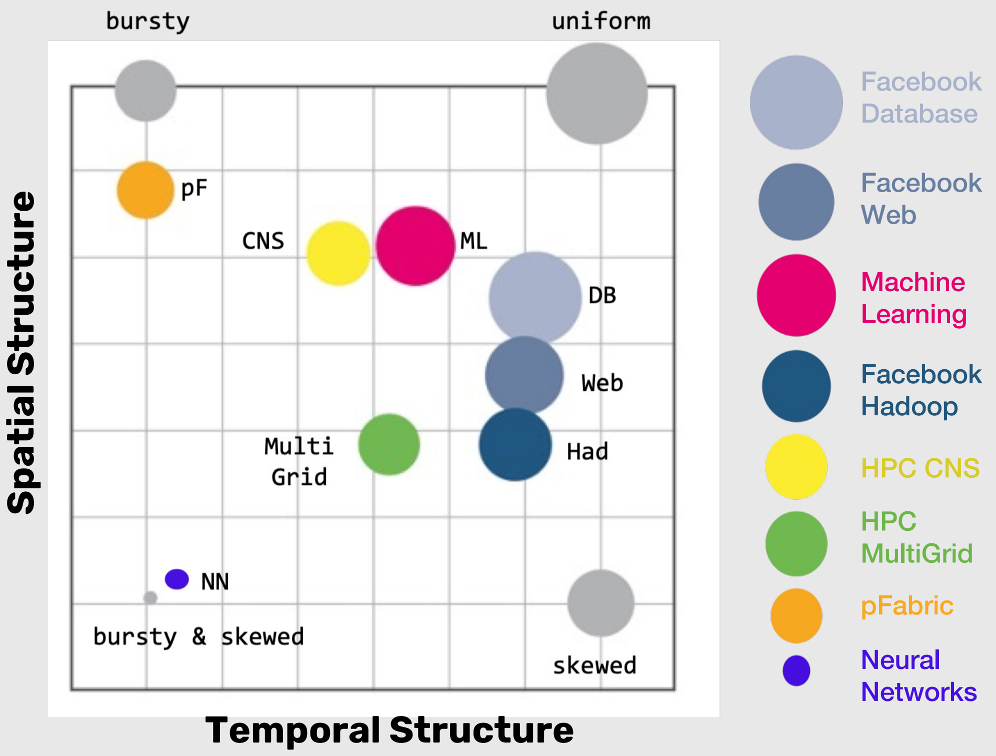
Structure in The Demand

Can we design a self-adjusting network that utilizes demand?

Let us start by a dynamically optimal self-adjusting tree!



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SeedTree

1) Why? 2) What? 3) How?

Requests From A Single Source

Using Local Routing (i.e., Without A Routing Table)!

Considering A Binary Tree Structure on Servers

Allowing Item Movements on Edges

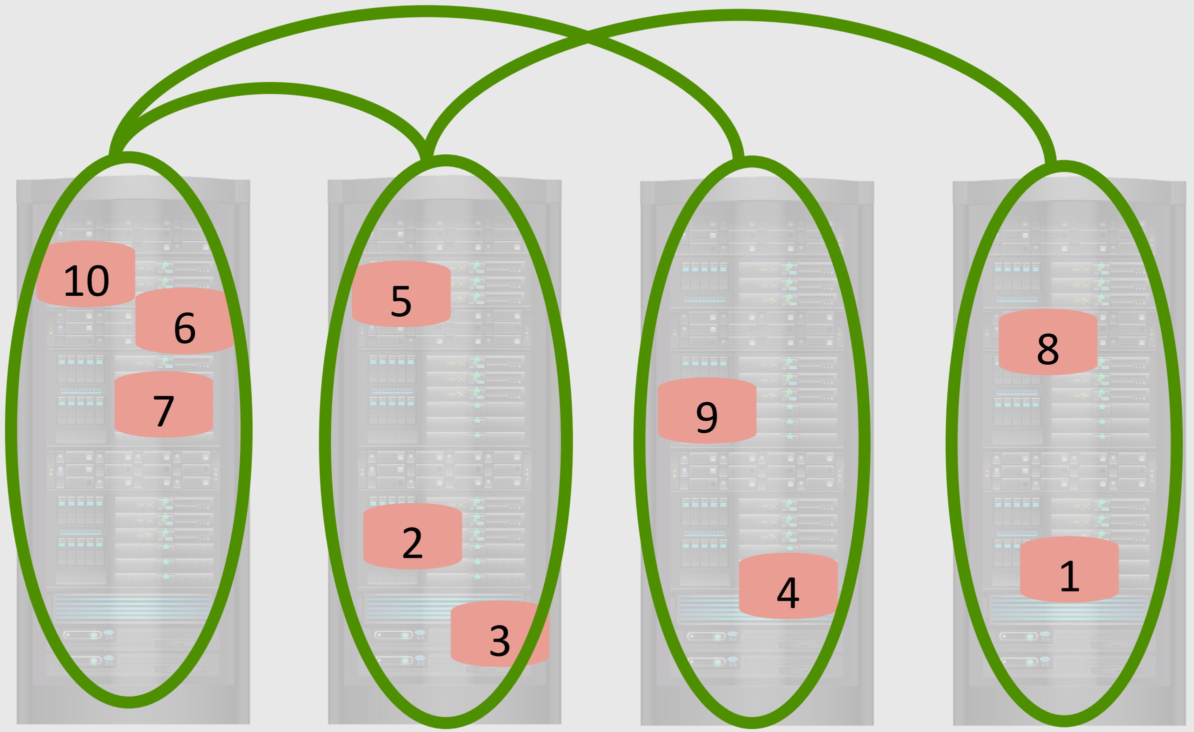
Allowing Item Movements on Edges

I do not have capacity!

Allowing Item Movements on Edges

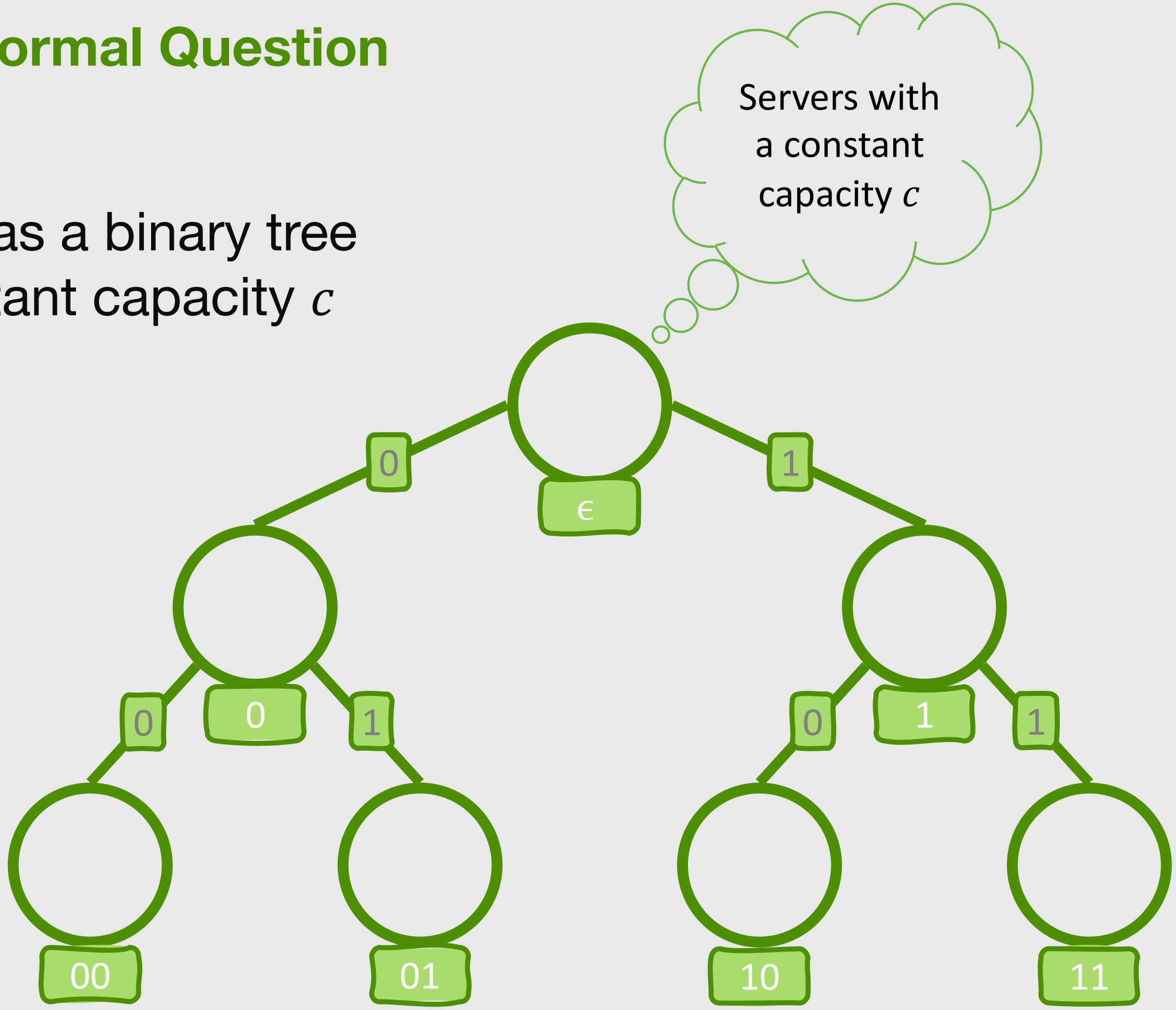
Allowing Item Movements on Edges

An Abstraction



Formal Question

- Given:
  - A set of servers connected as a binary tree
    - Each server with a constant capacity c



Tree

Formal Question

- Given:
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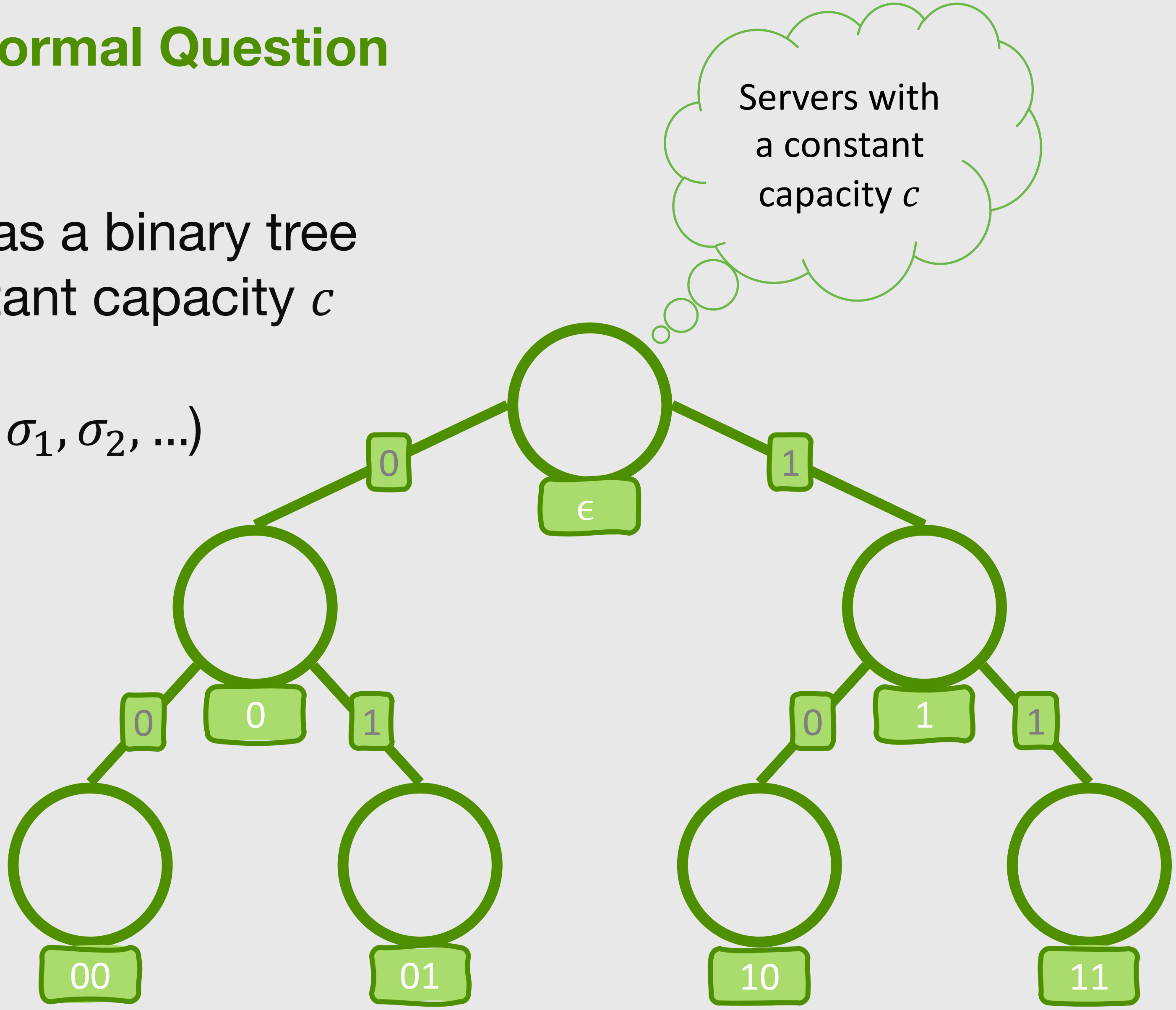
  - A set of items
    - Revealed over time ($\sigma = \sigma_1, \sigma_2, \ldots$)

Items



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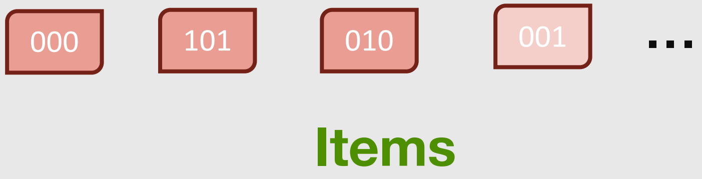

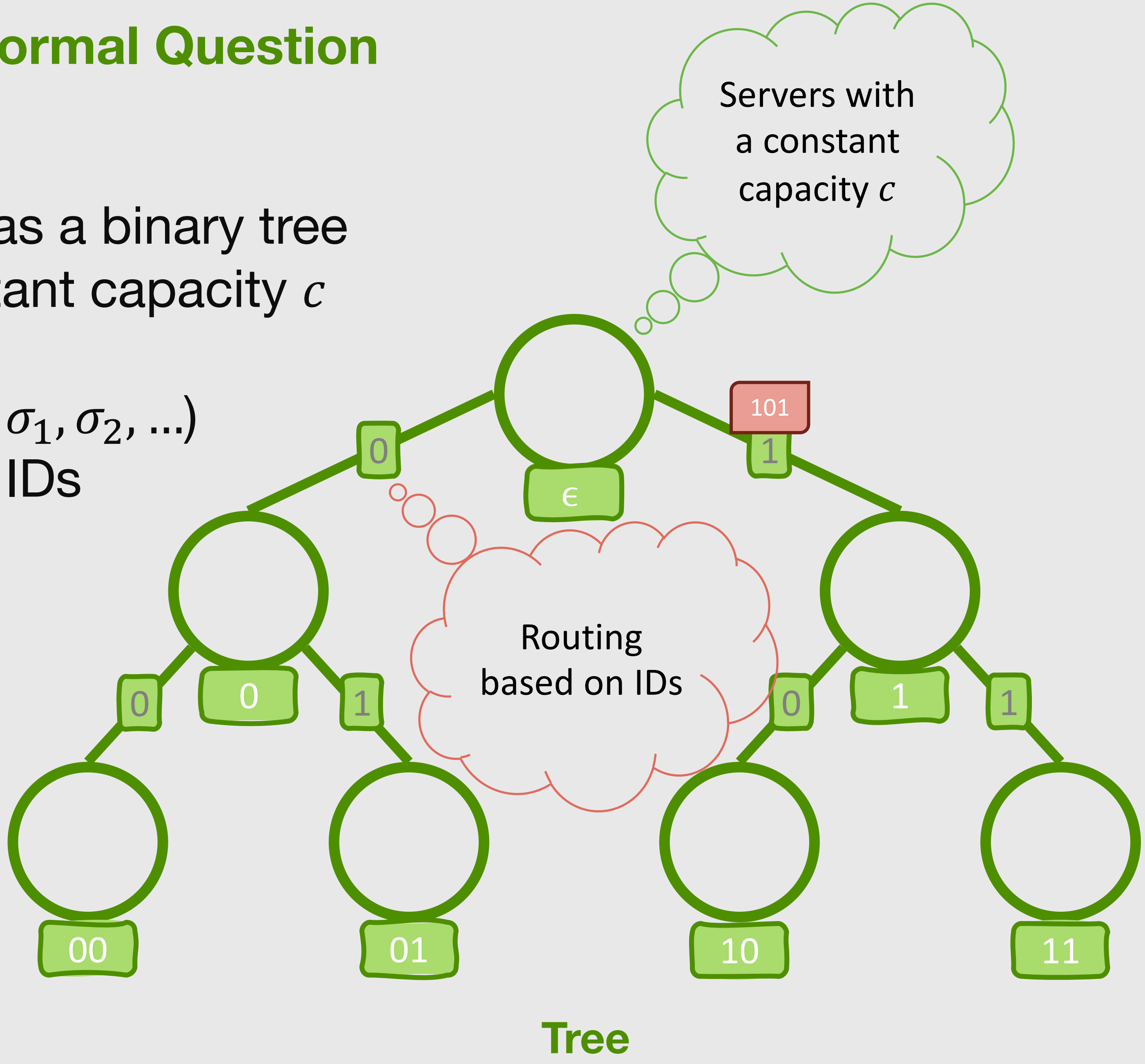

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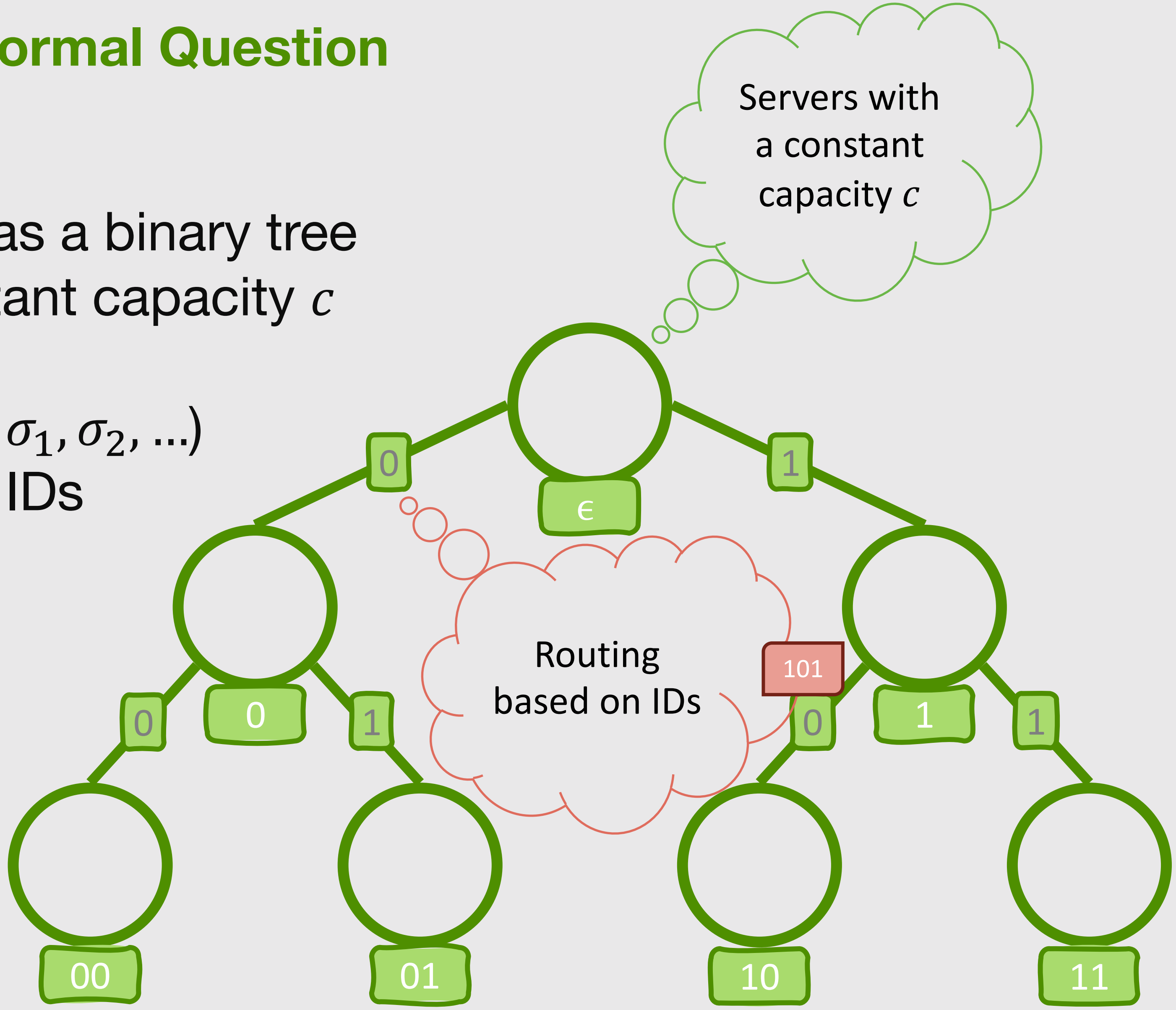

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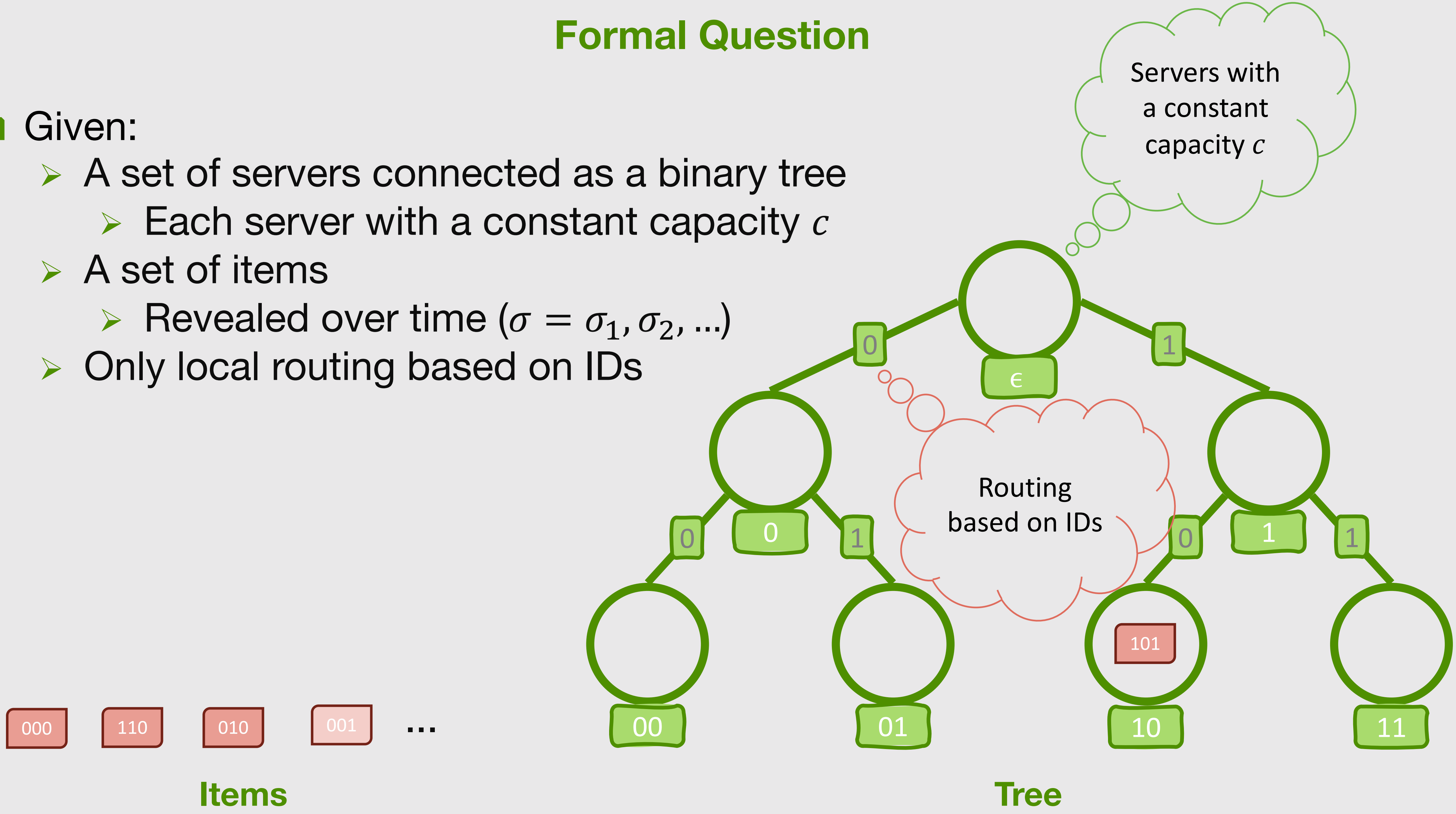

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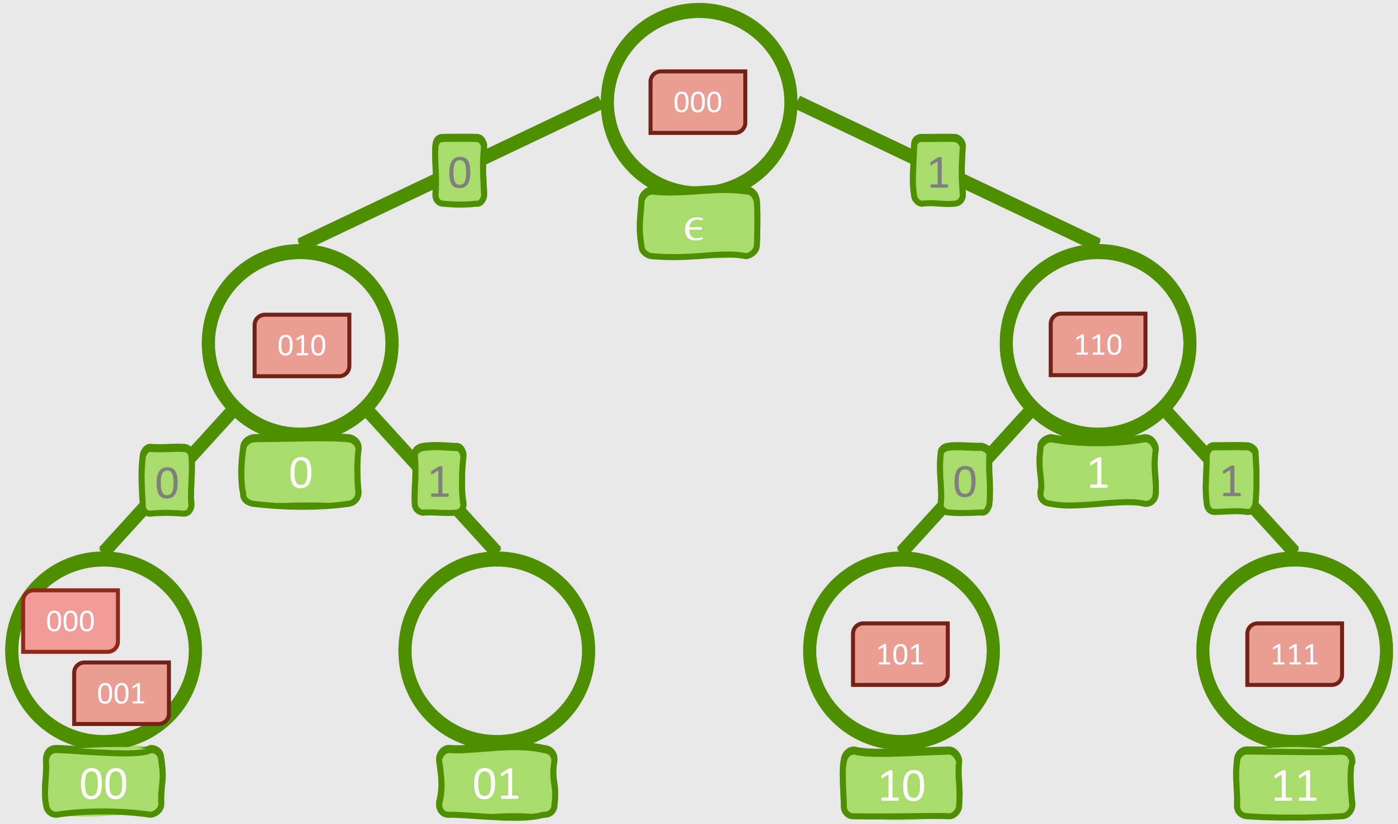
- Actions (for a prefilled tree*):



Formal Question

- Actions (for a prefilled tree*):
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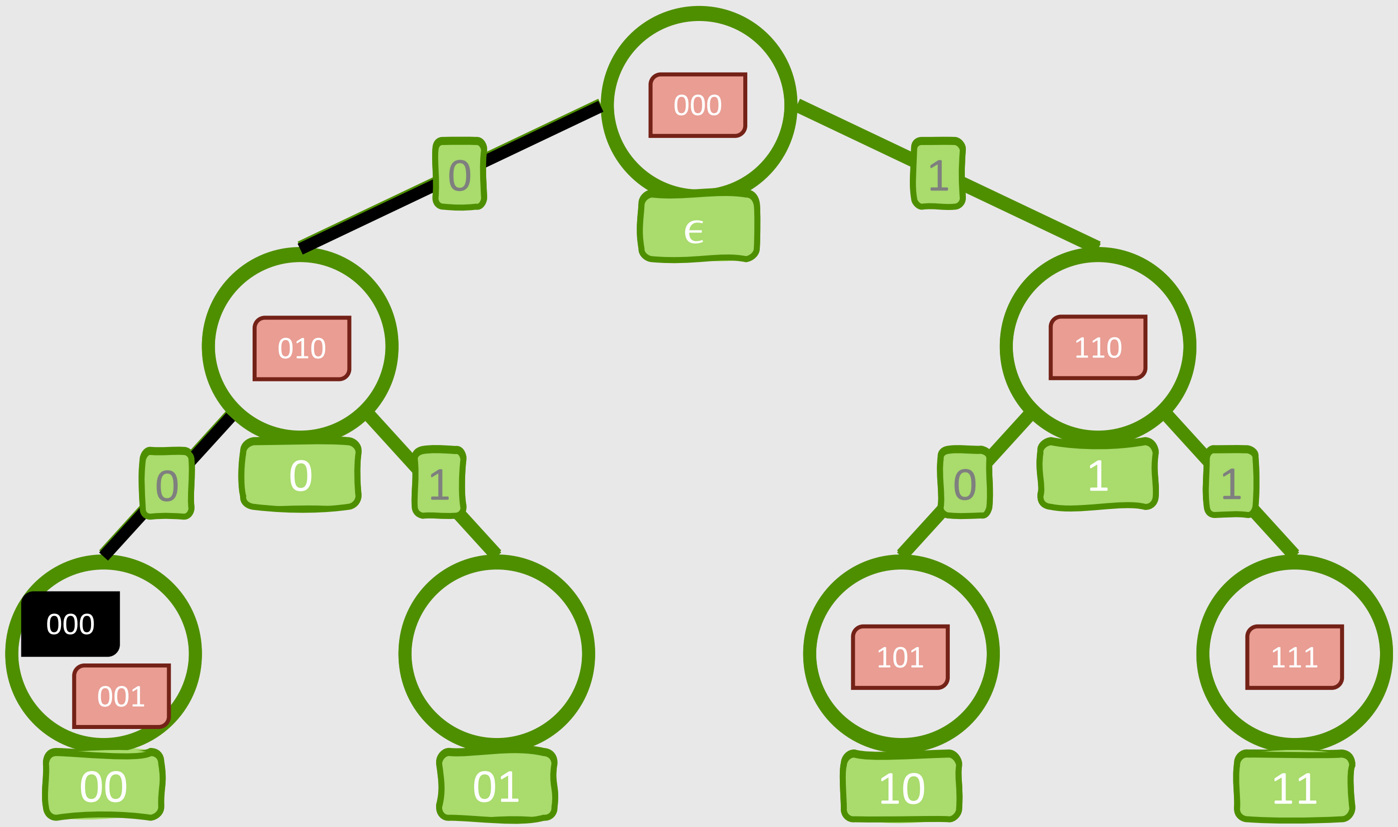

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- Actions (for a prefilled tree*):
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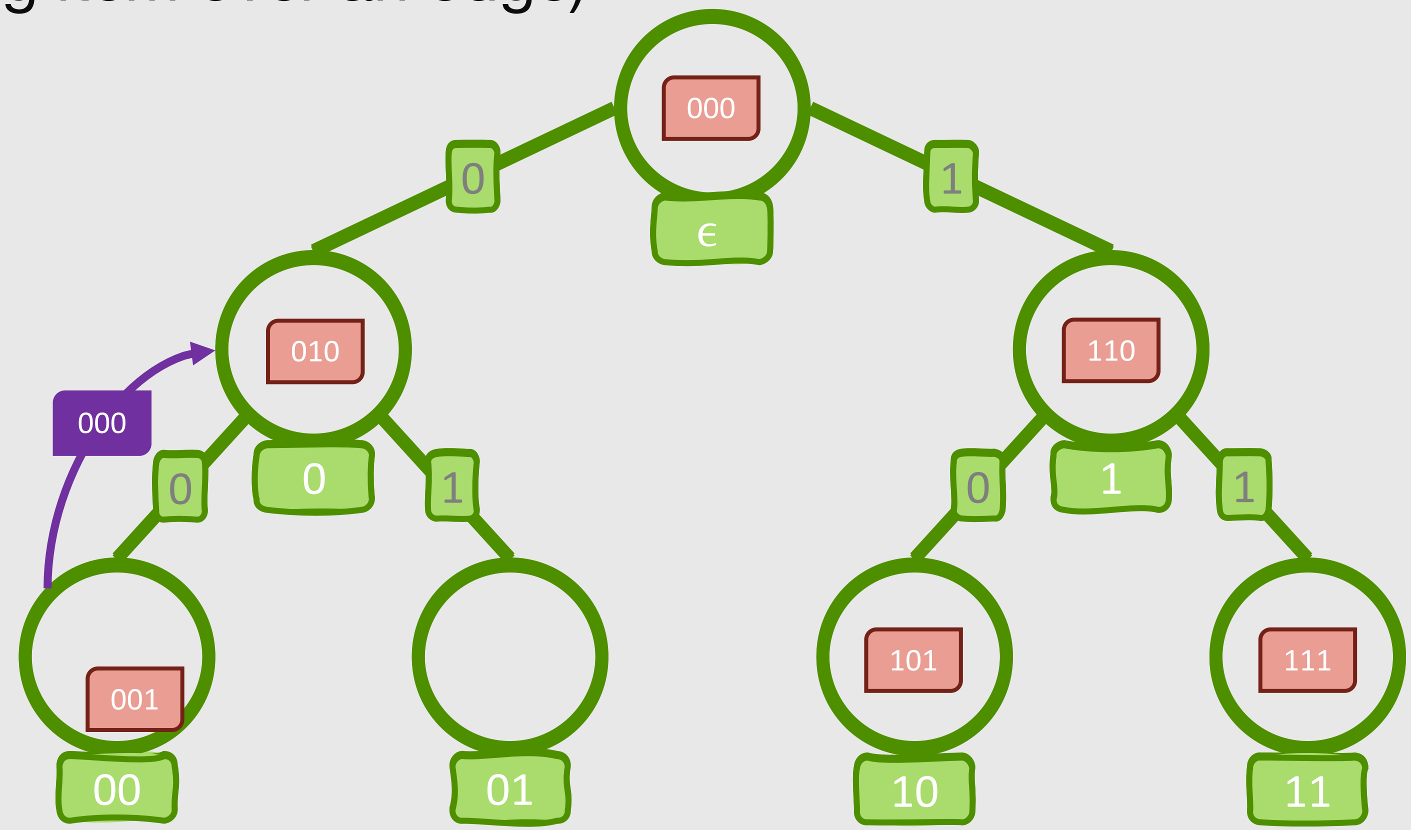

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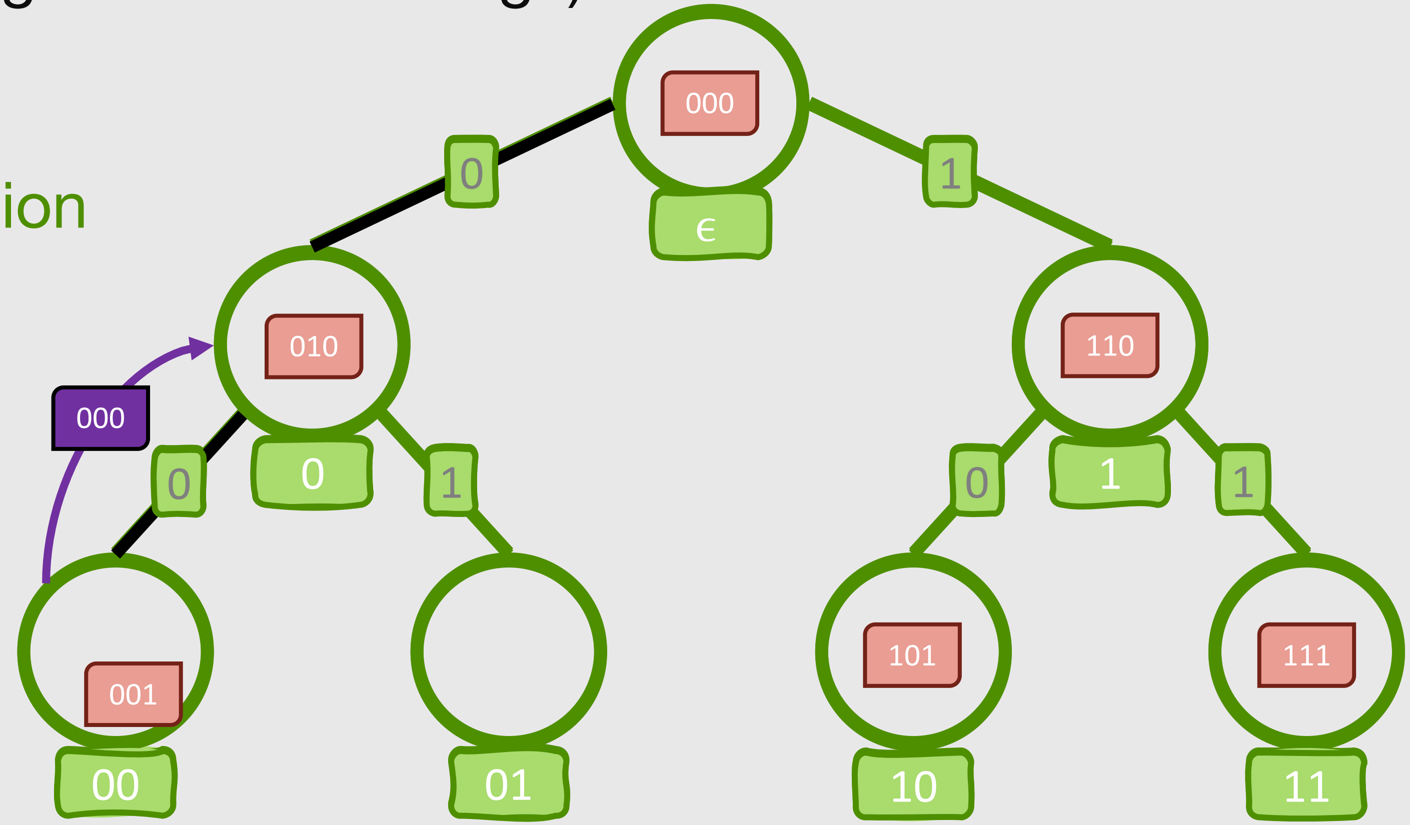
- Objective
  - Minimize the total cost
  - Total: access + reconfiguration



Formal Question

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  - Access an element (depth)
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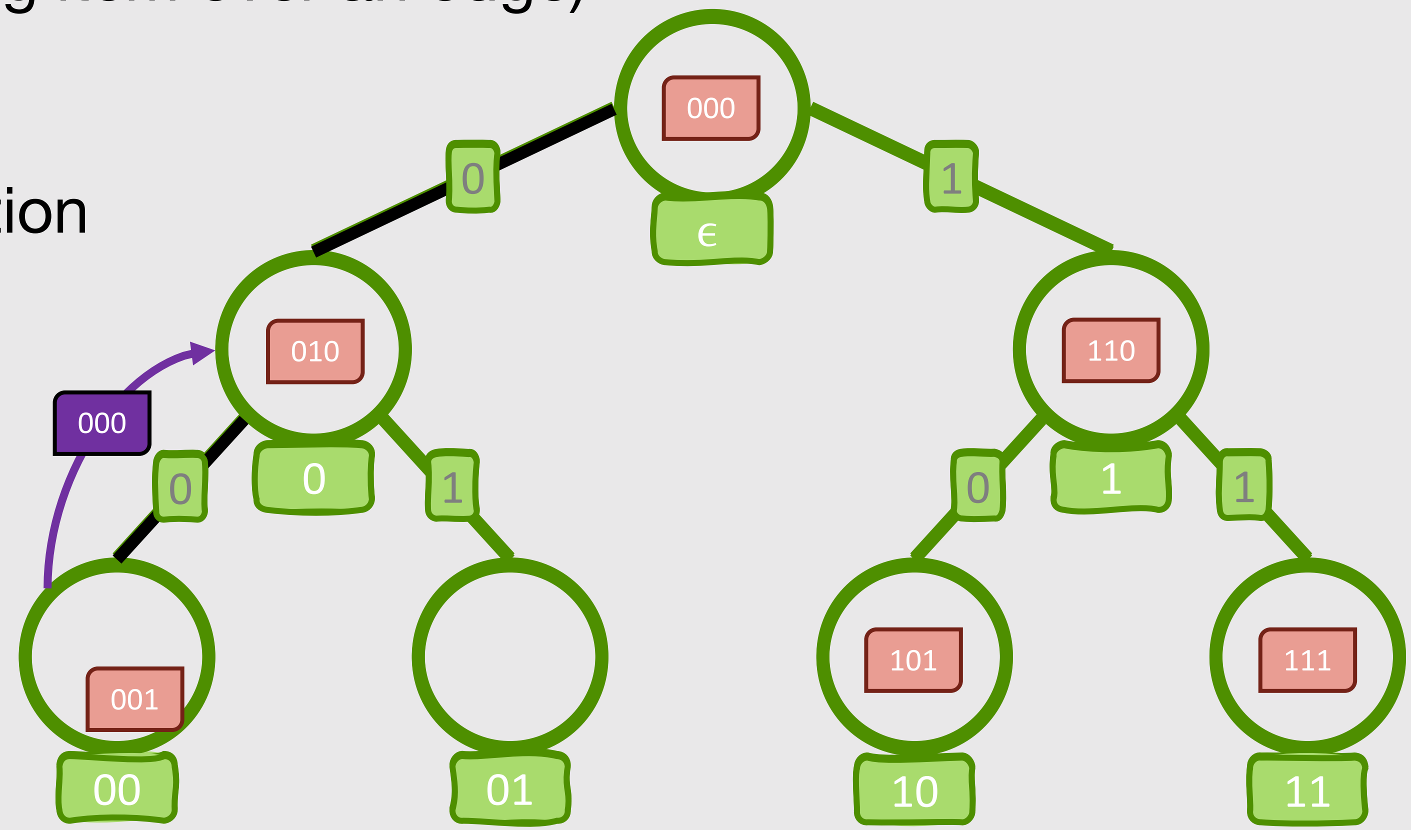
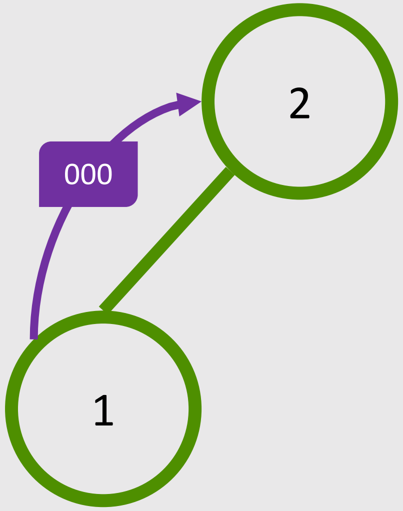
  - Total: access + reconfiguration
- Dynamically optimal
  - i.e., constant competitive
  - $Cost_{ALG} \leq \alpha \cdot Cost_{OPT}$



Previous work

Data structure	Operation	Dynamically Optimal	Local Routing?
SeedTree [Pouradmghani et al., INFOCOM'23]	Item Movement	✅	✅



Moving
Items

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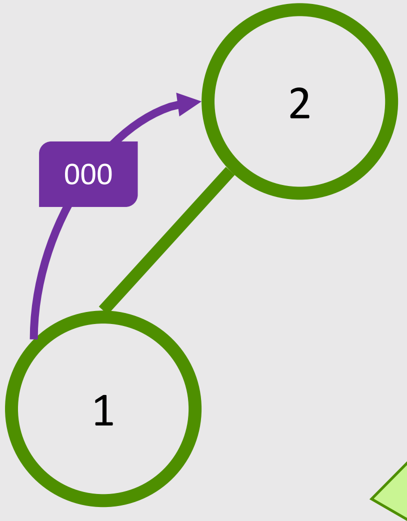
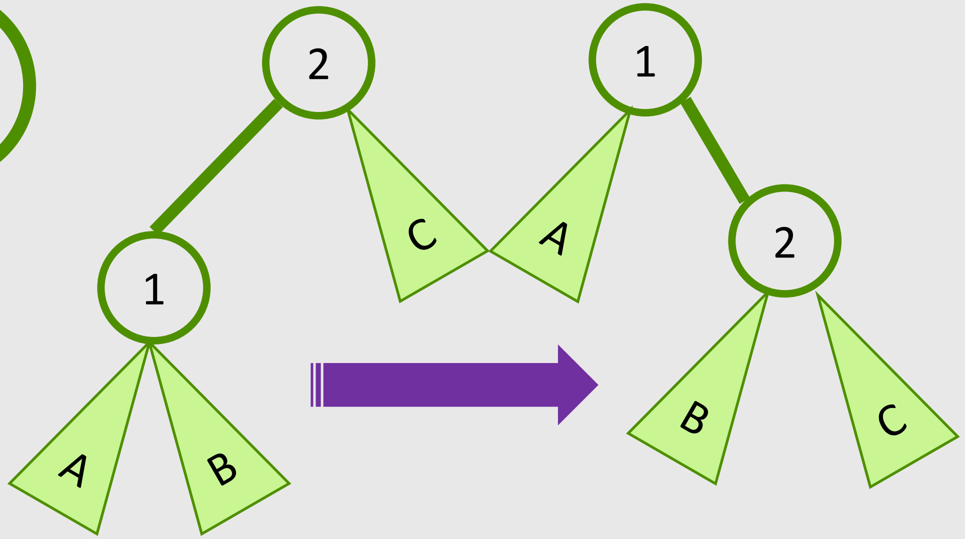
Tango Tree [Demaine et al. FOCS'04, J. Comput07]	Rotation	❌	✅
MultiSplay [Wamg et al. SODA'06]	Rotation	?	✅



Moving Items



Rotation

Previous work

Data structure	Operation	Dynamically Optimal	Local Routing?
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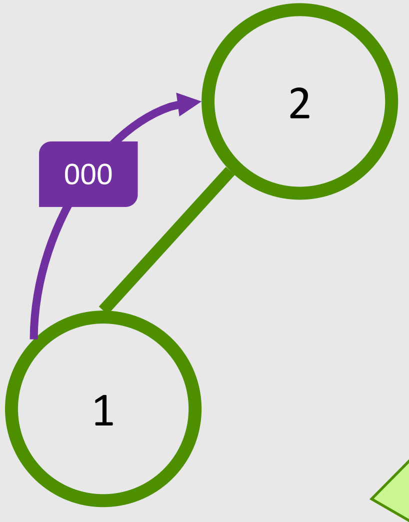
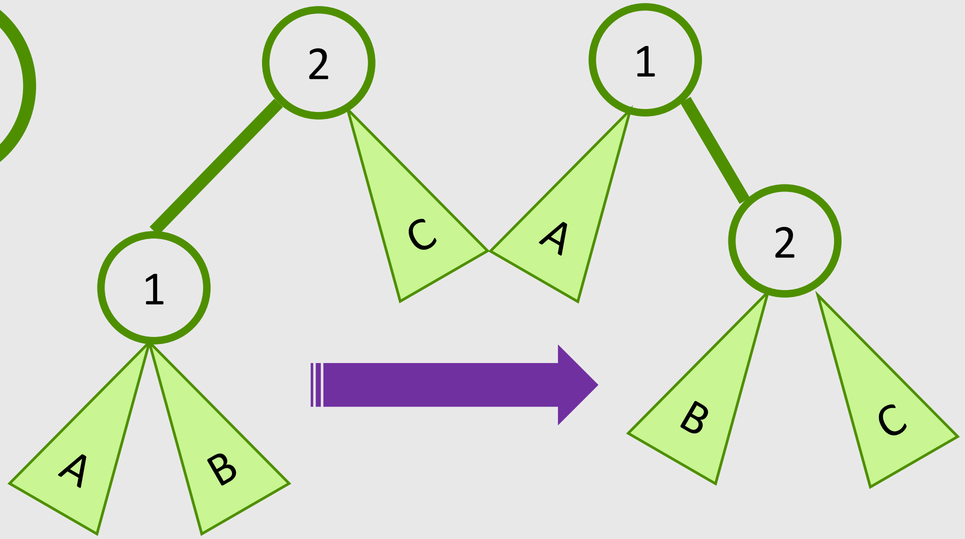
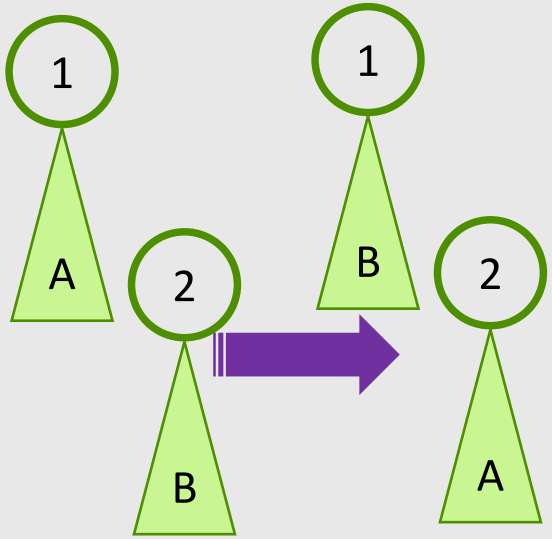
Adaptive Huffman [Vitter J.ACM’87, FGK J.ACM’85]	Subtree Swap	✅	❌



Moving Items



Rotation



Subtree Swap

Previous work

Data structure	Operation	Dynamically Optimal	Local Routing?
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Splay Tree [Sleator & Tarjan J. ACM'85]	Rotation	?	✅
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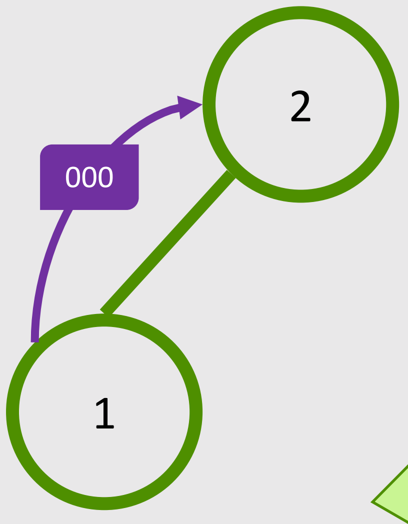
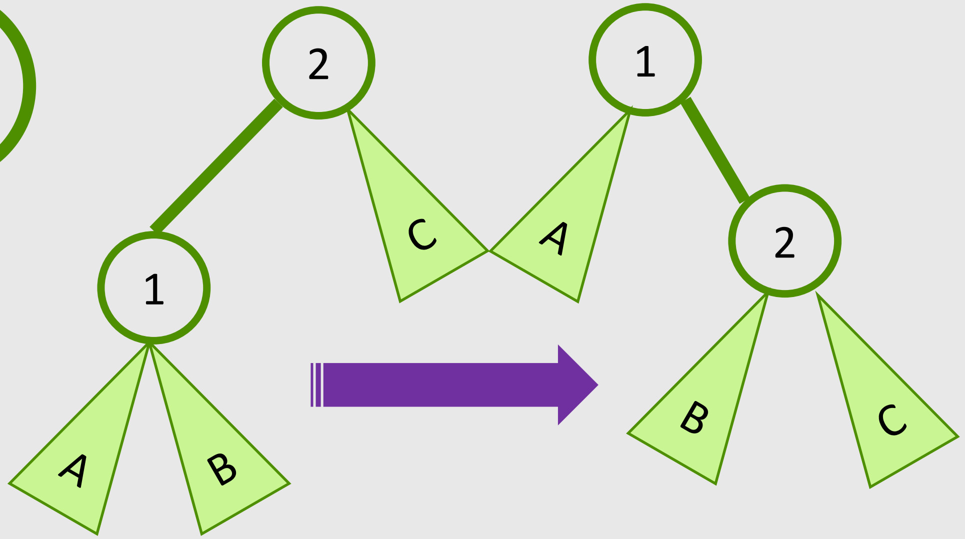
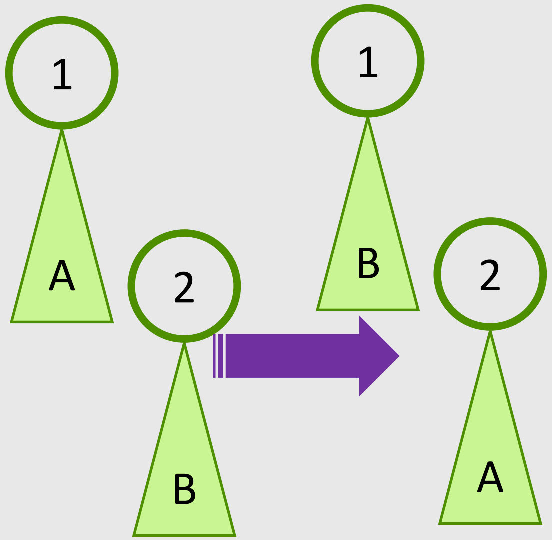
MultiSplay [Wamg et al. SODA'06]	Rotation	?	✅
Adaptive Huffman [Vitter J.ACM'87, FGK J.ACM'85]	Subtree Swap	✅	❌
Push-down-Tree [Avin et al. LATIN'20, TON'22]	Item Swap	✅	❌



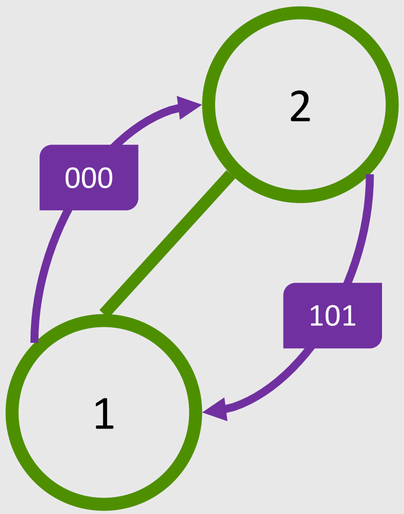
Moving Items



Rotation



Subtree Swap

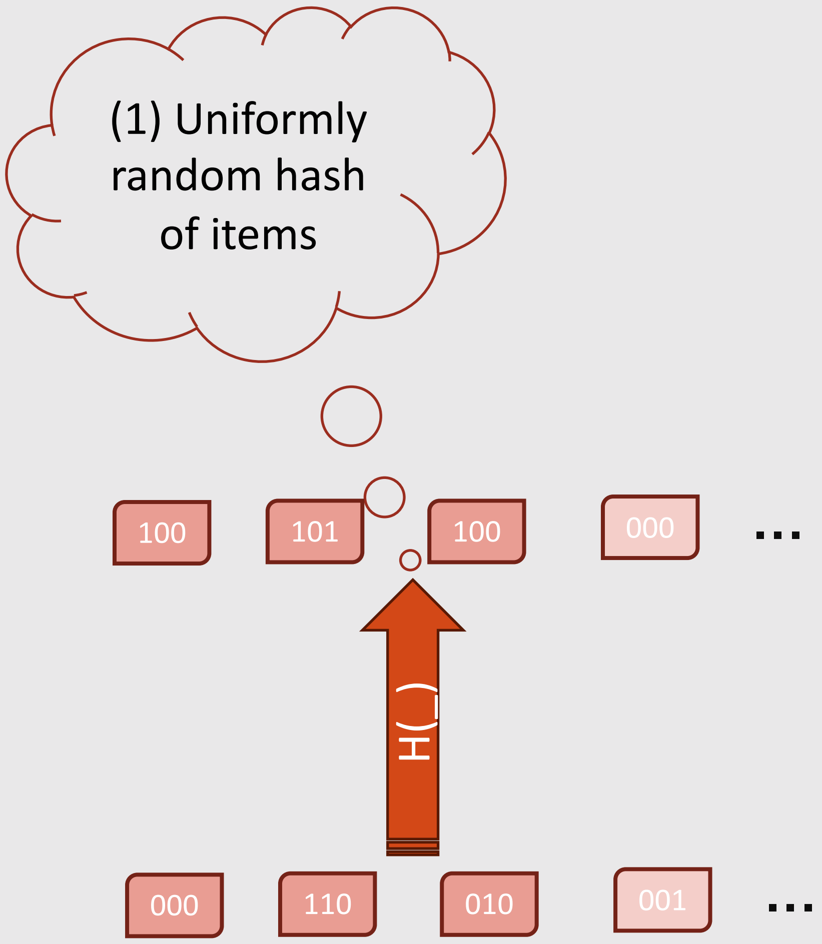


Item Swap

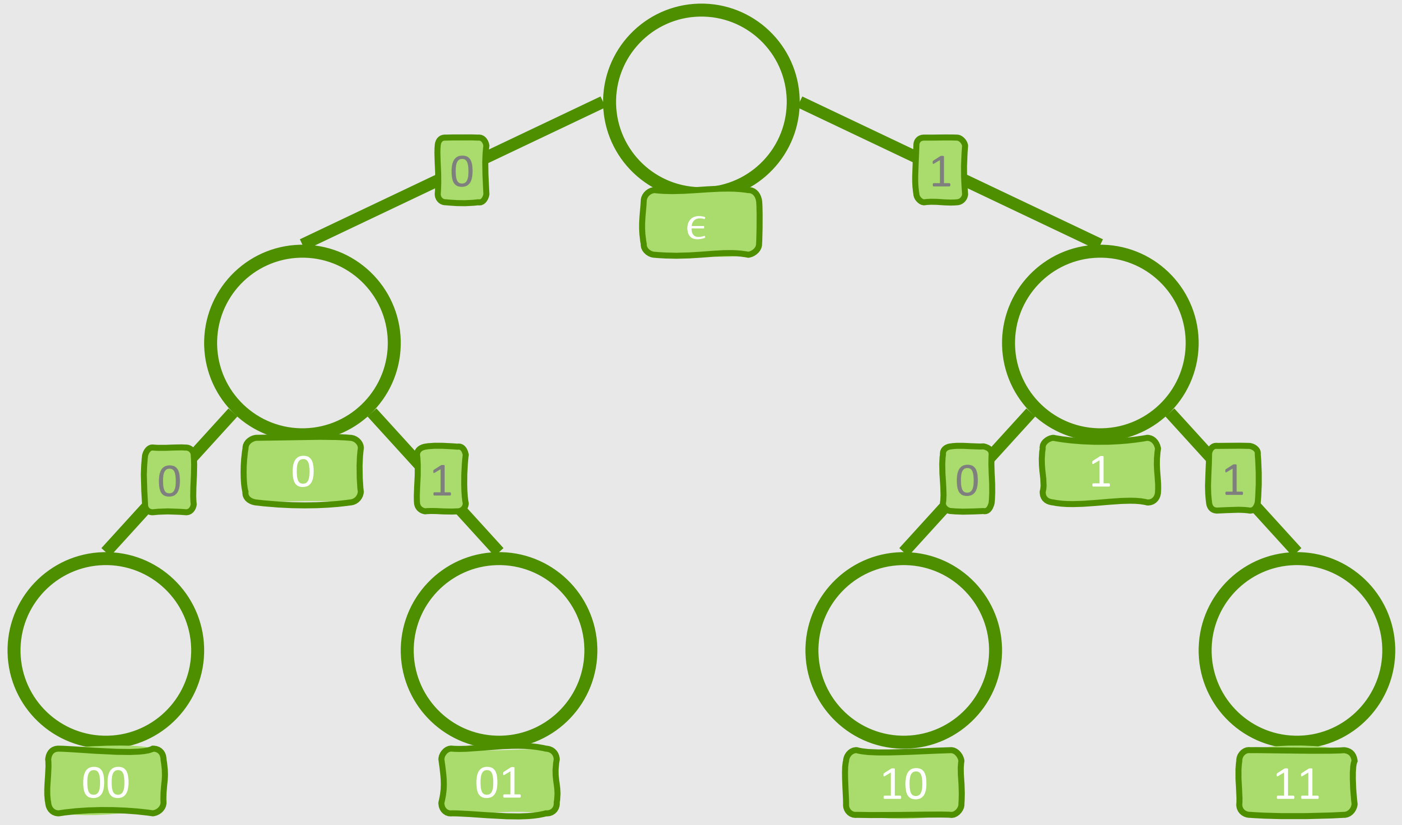
SeedTree

1) Why? 2) What? 3) How?

Tools & Techniques



Items



Tree

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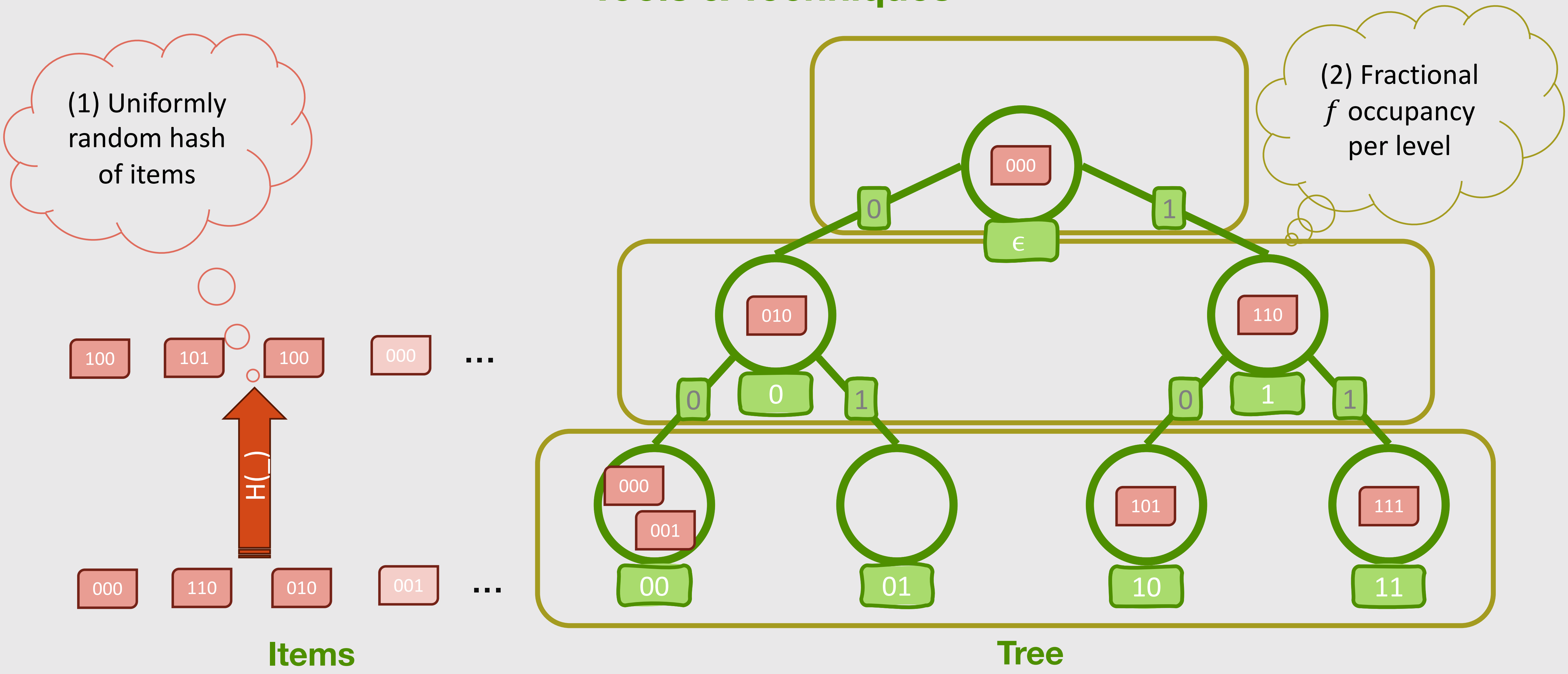
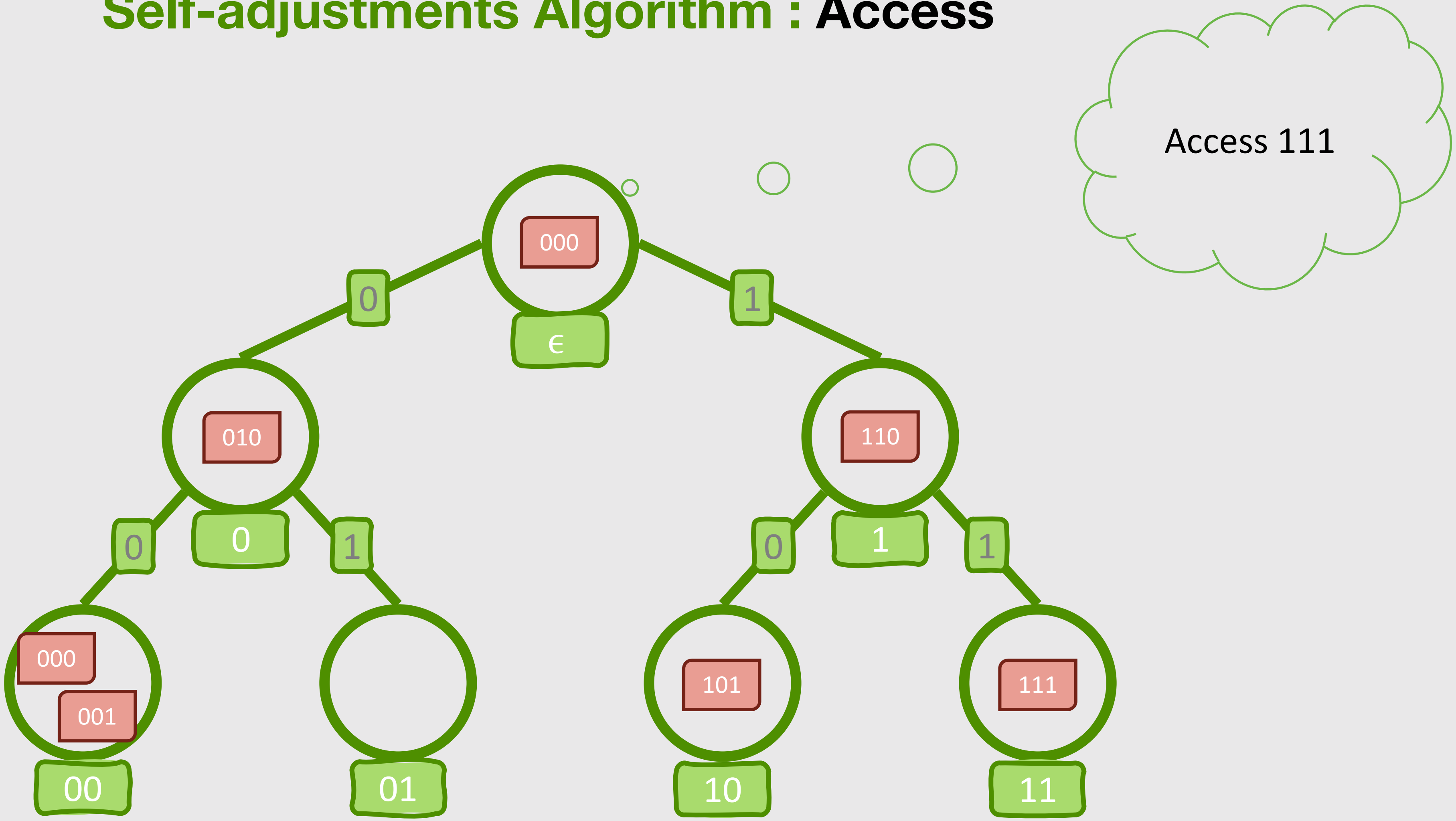


Self-adjustments Algorithm : Access



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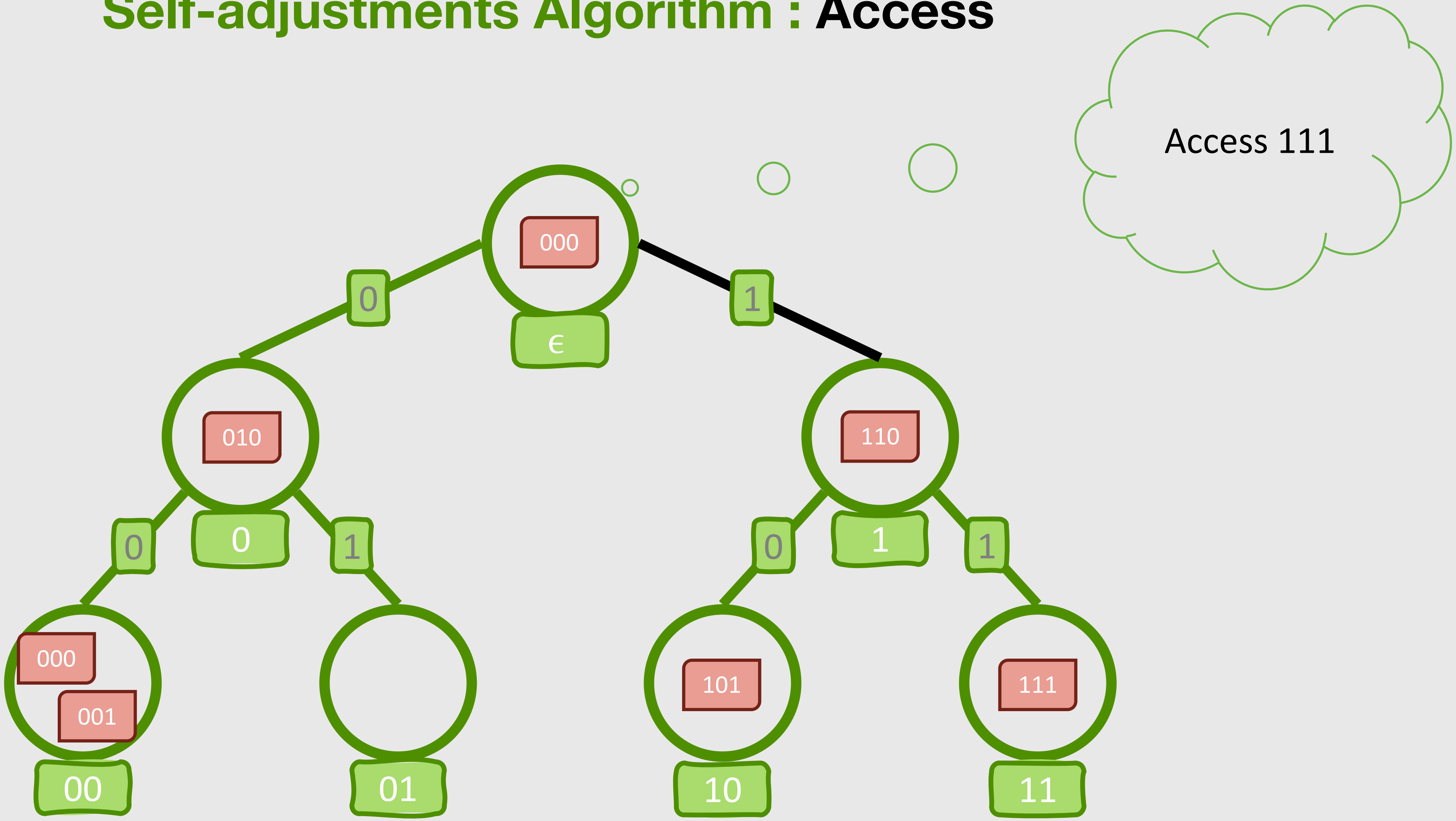

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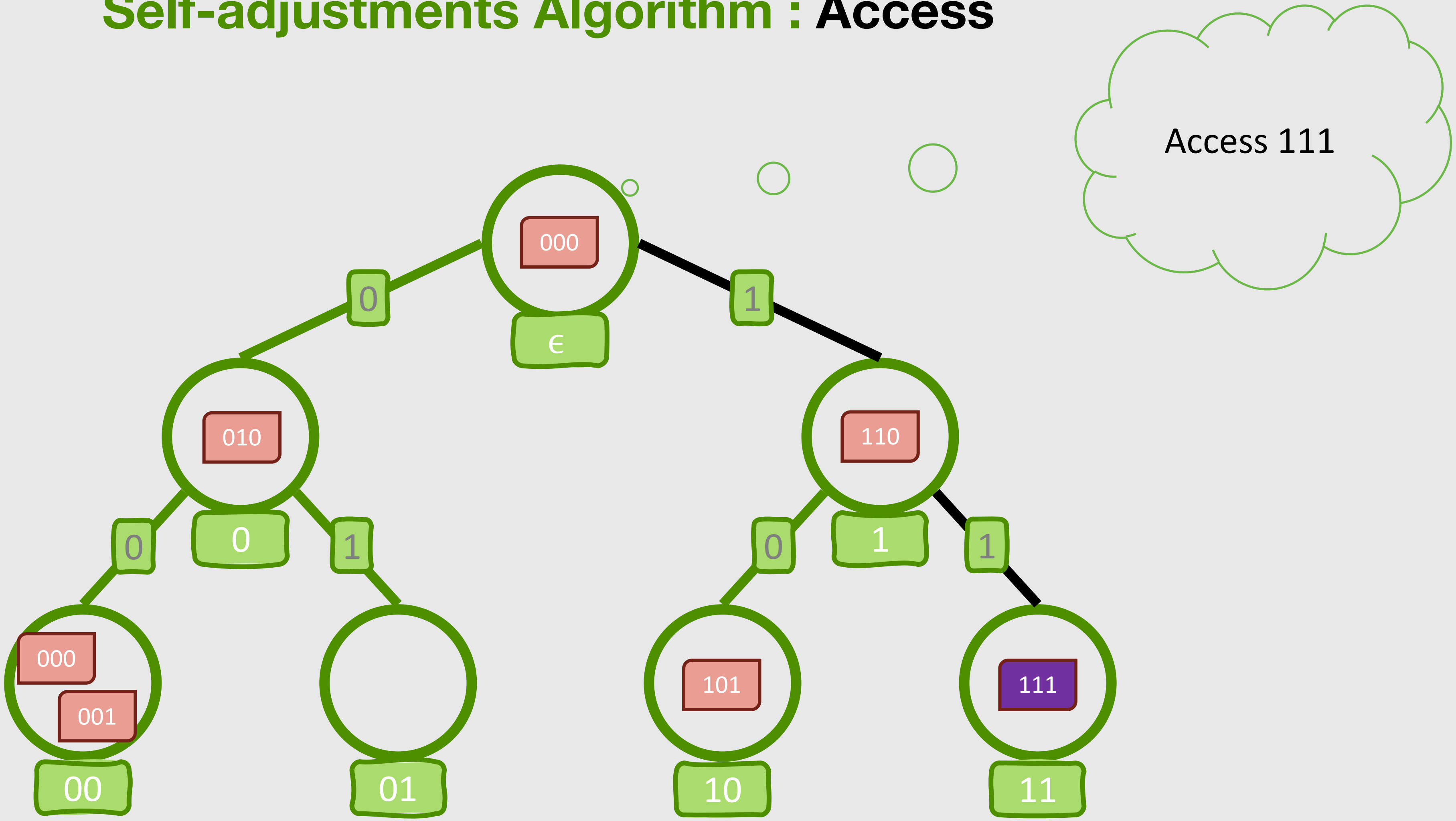
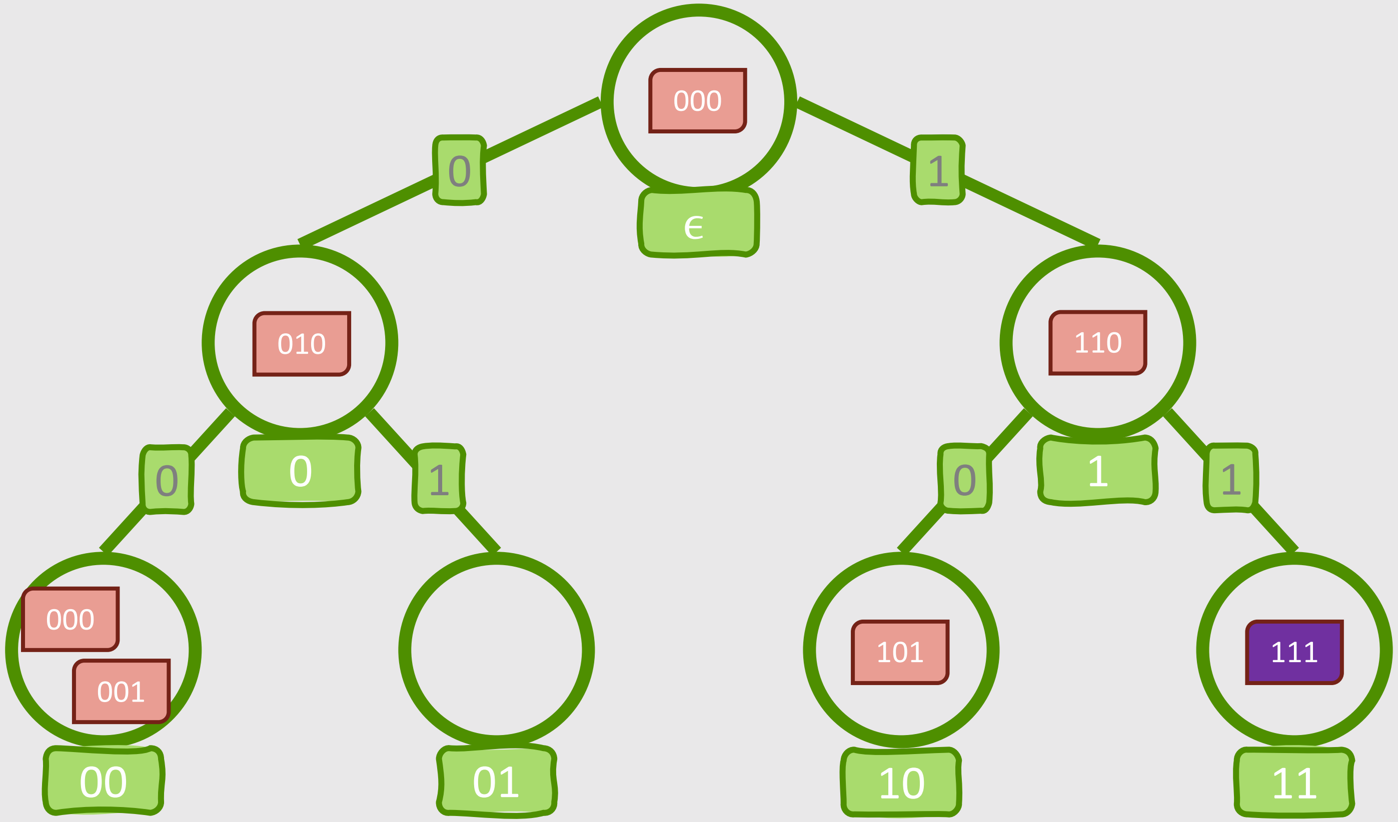


Self-adjustments Algorithm : **Pull-up**



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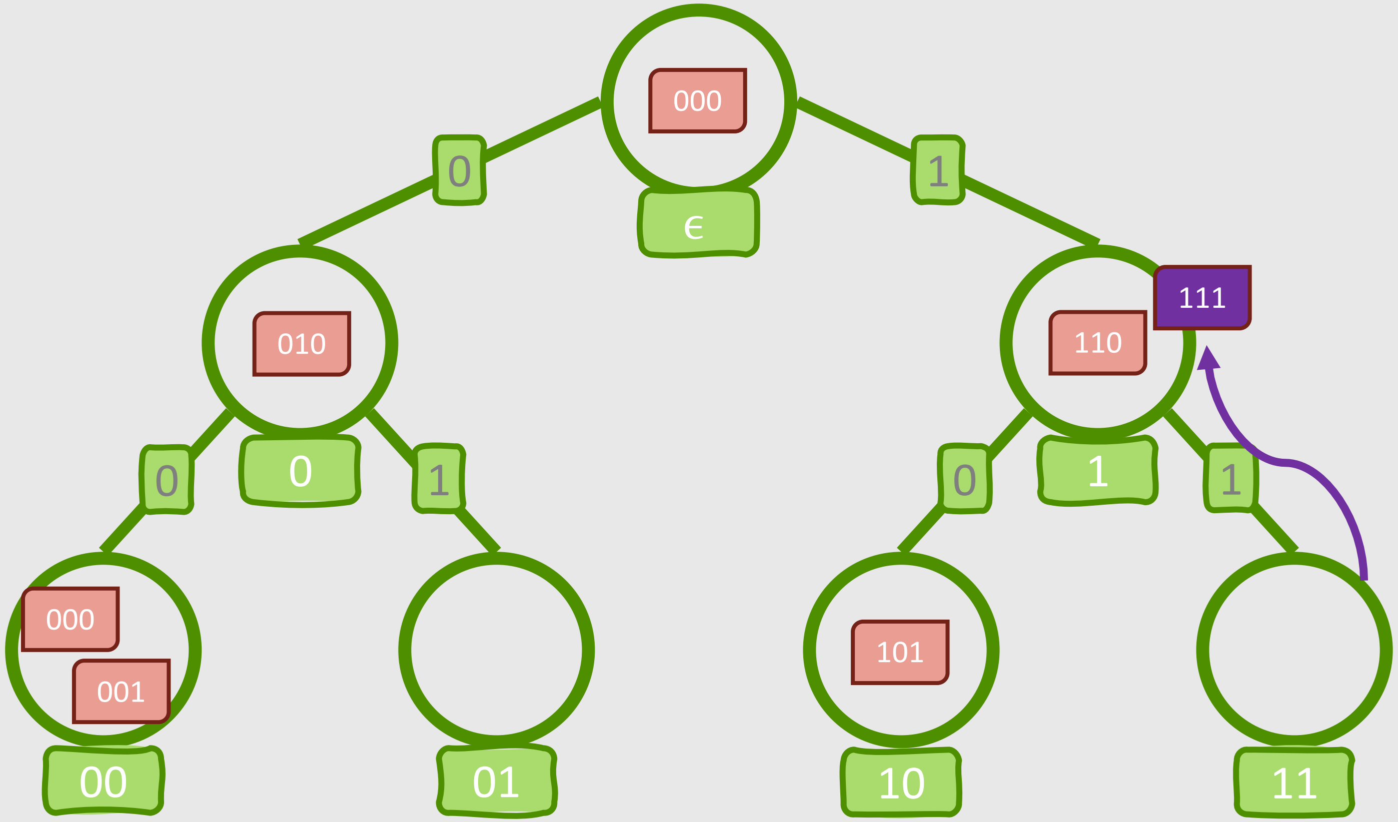

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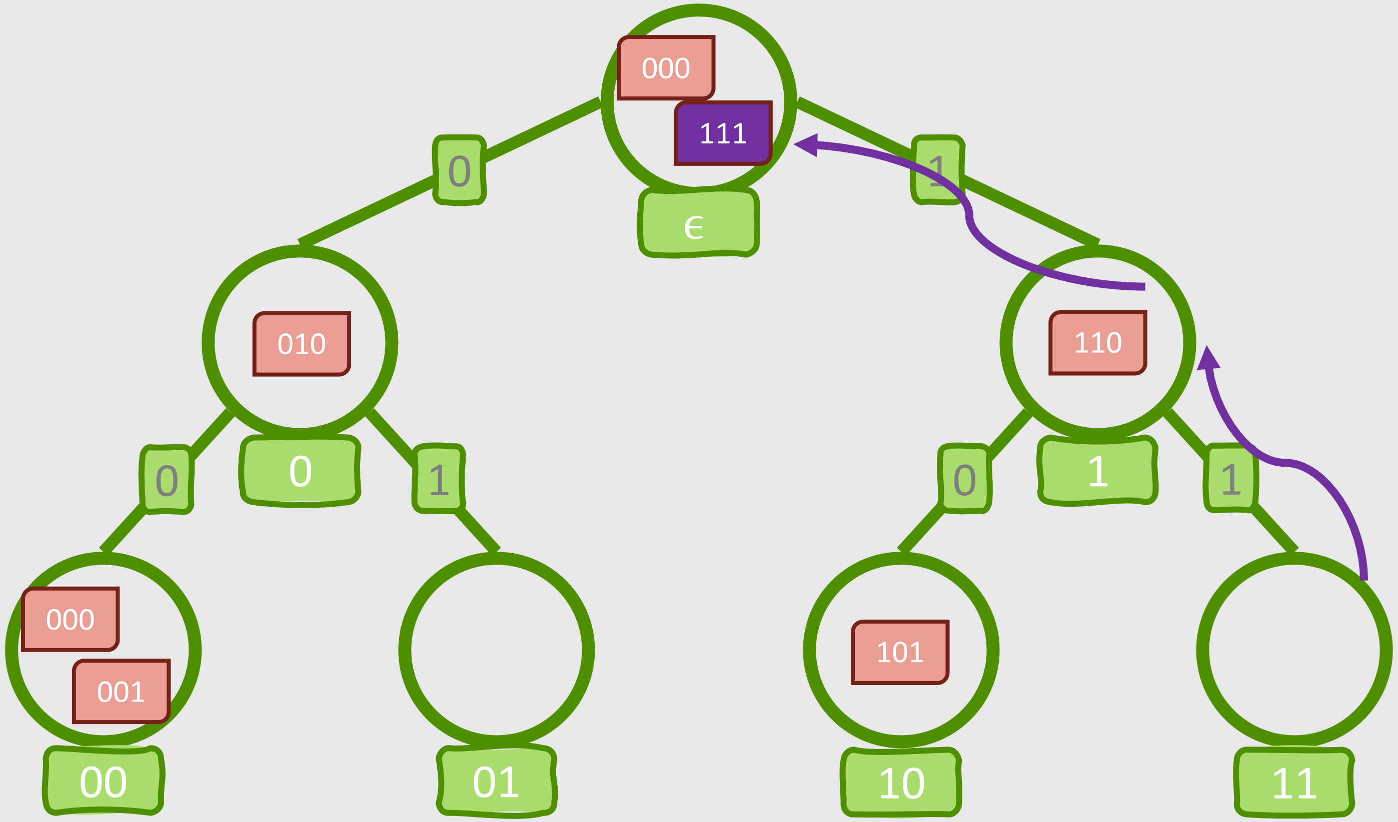
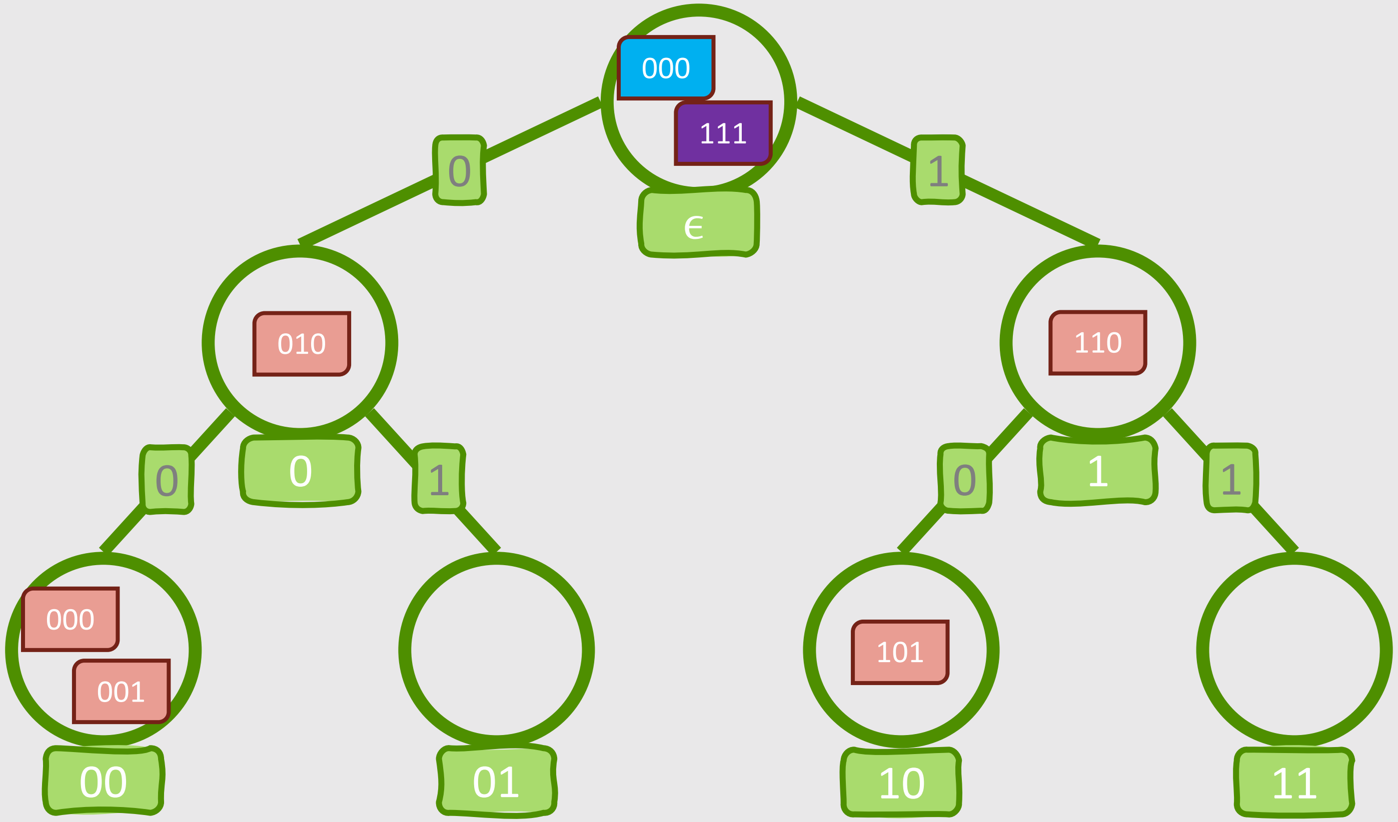


Self-adjustments Algorithm: Push-down



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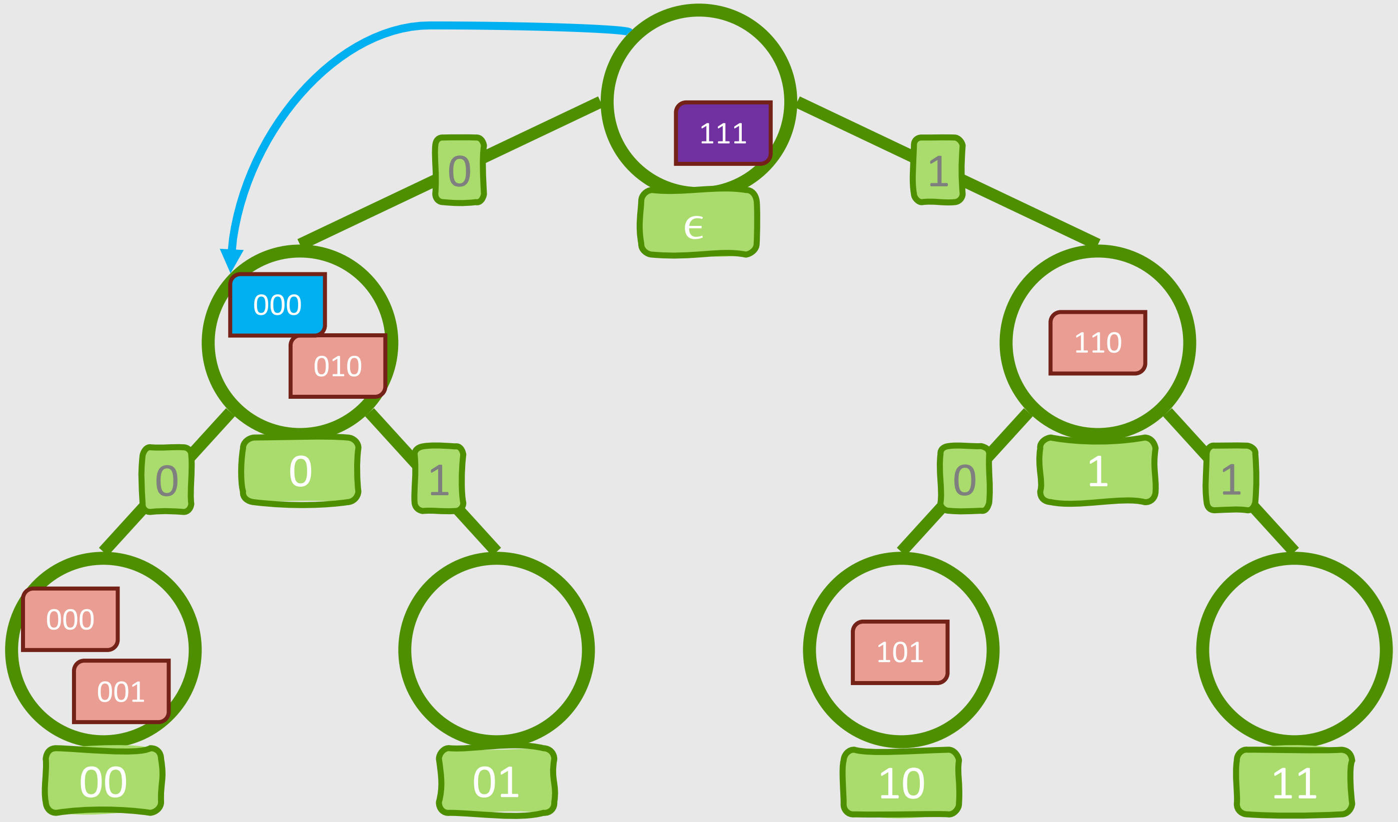

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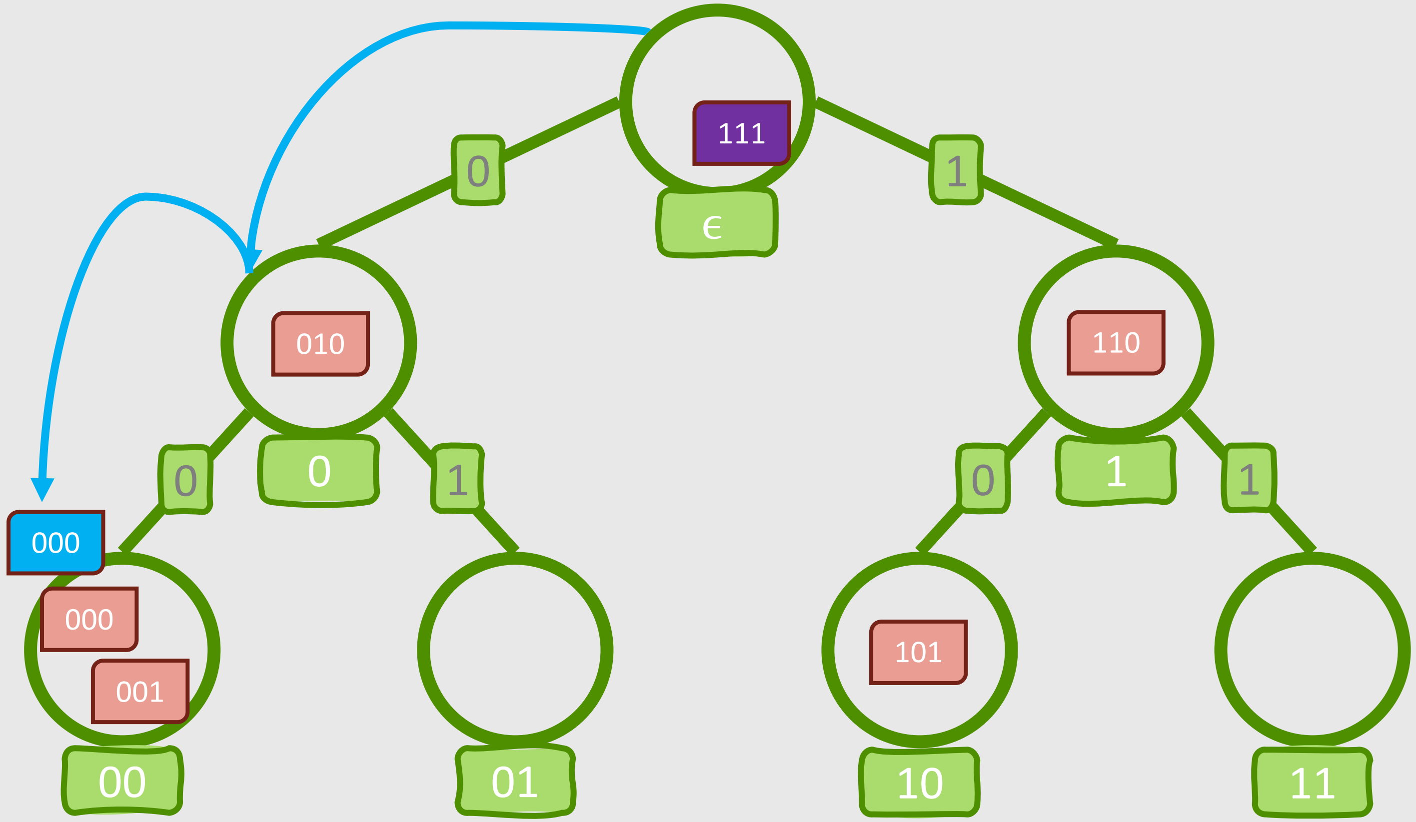

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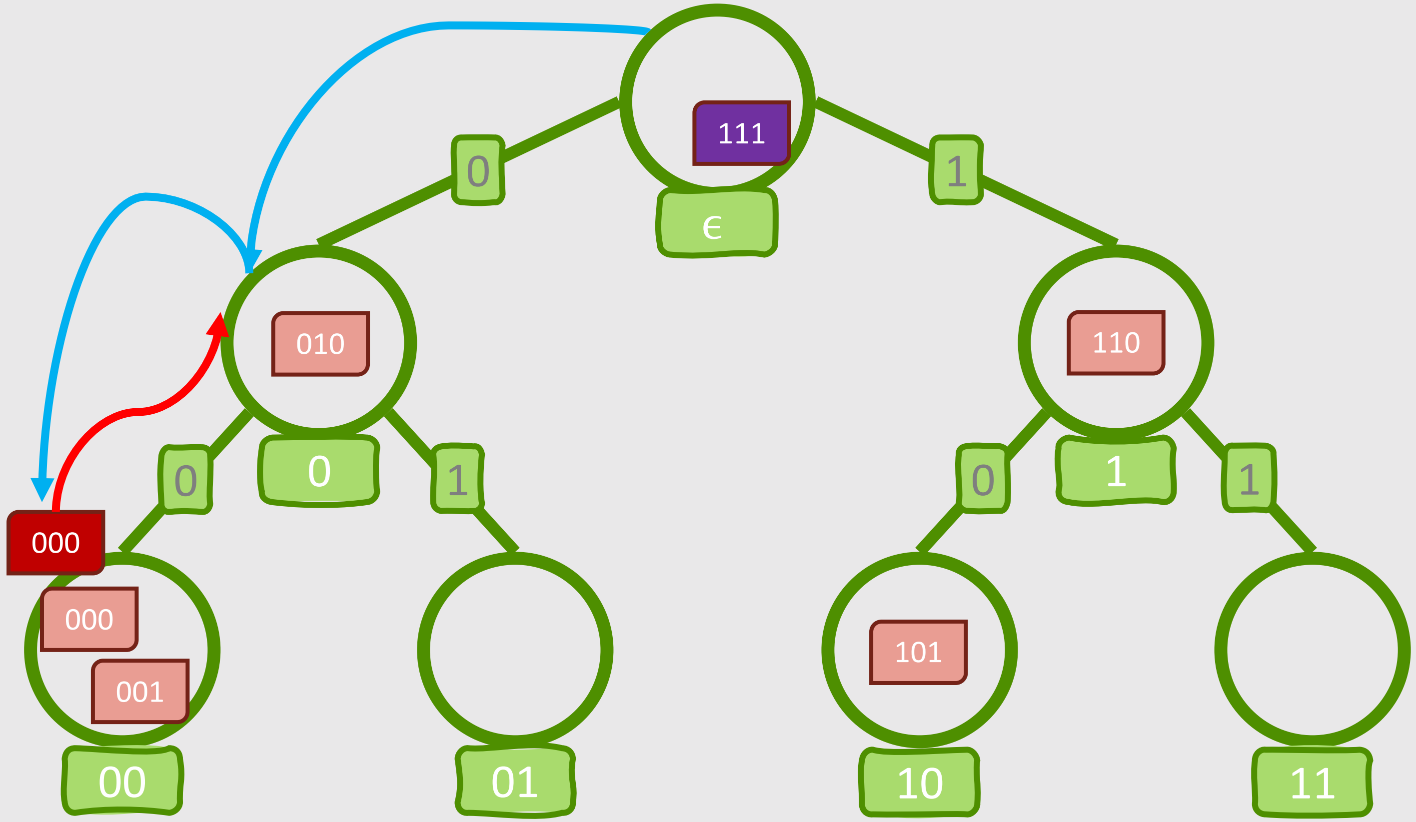

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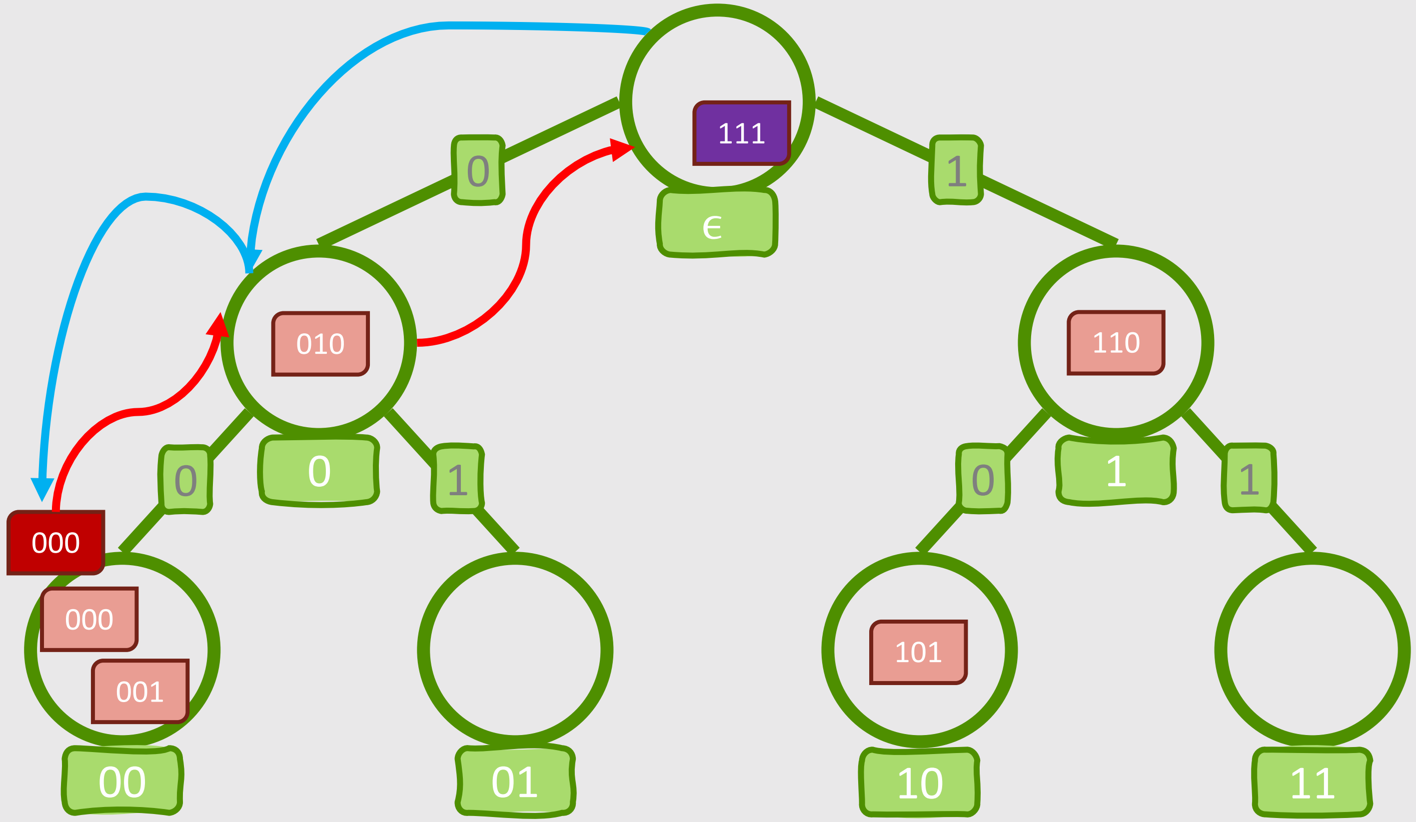

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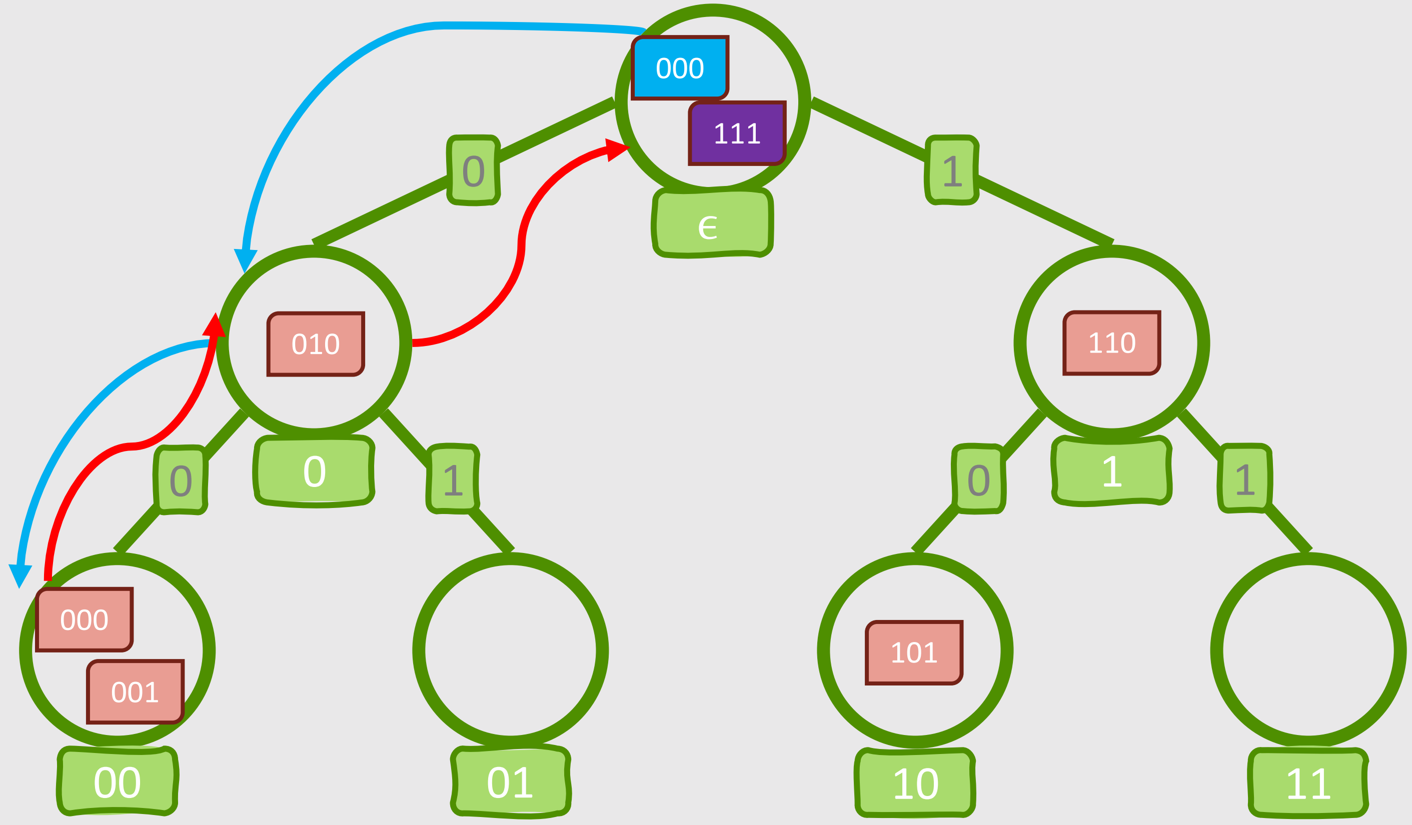

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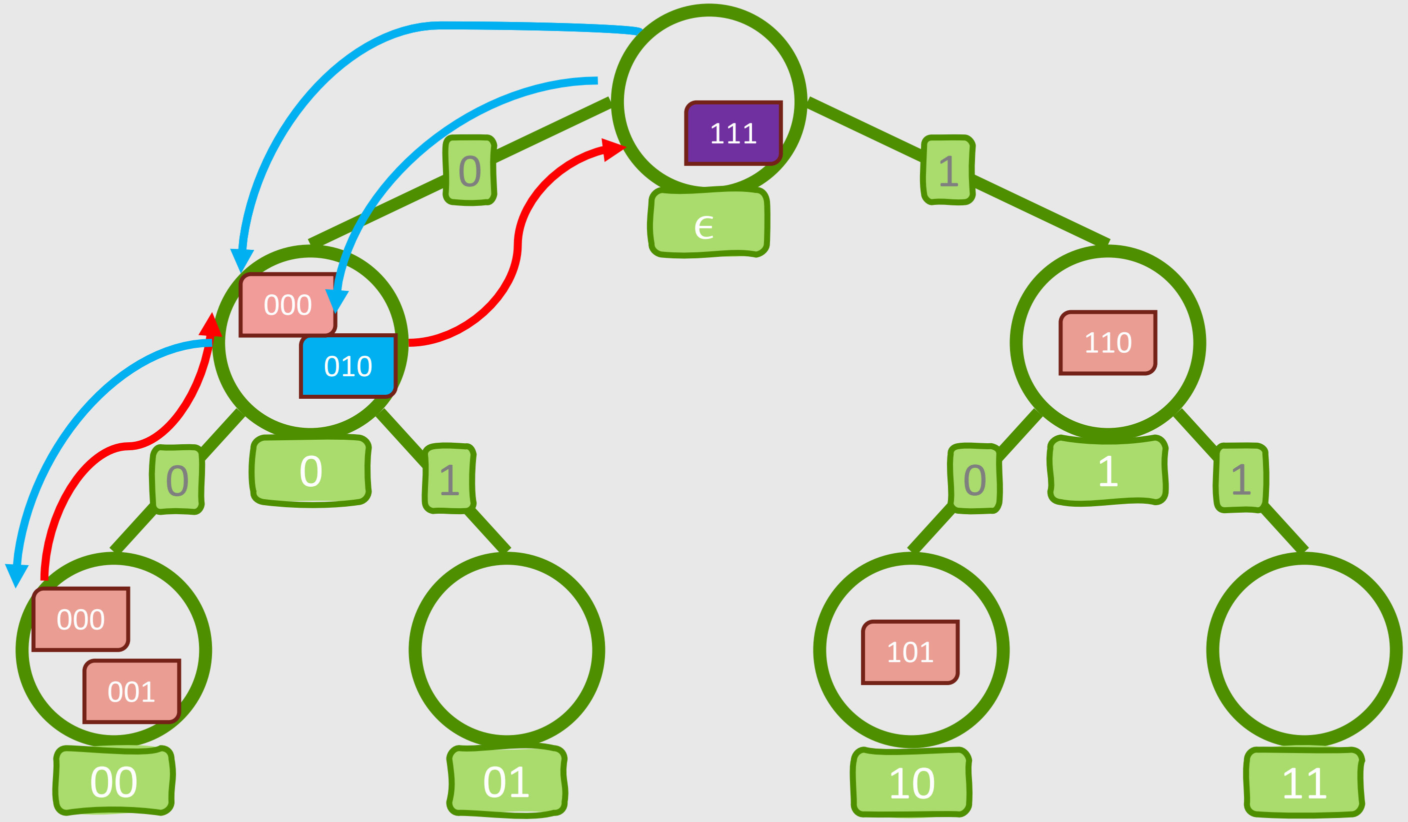

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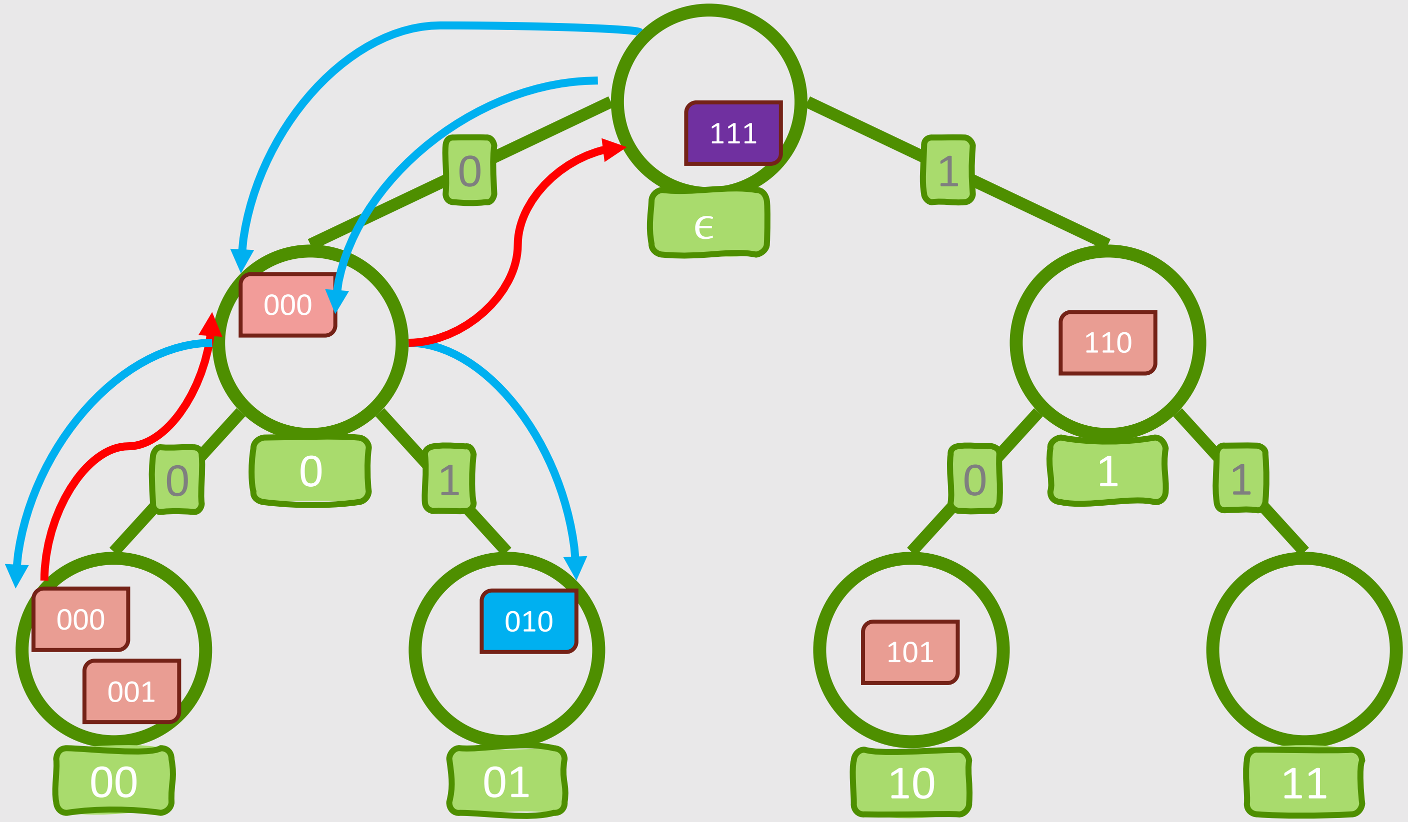

SeedTree Analysis

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- Objective (over time and in expectation):

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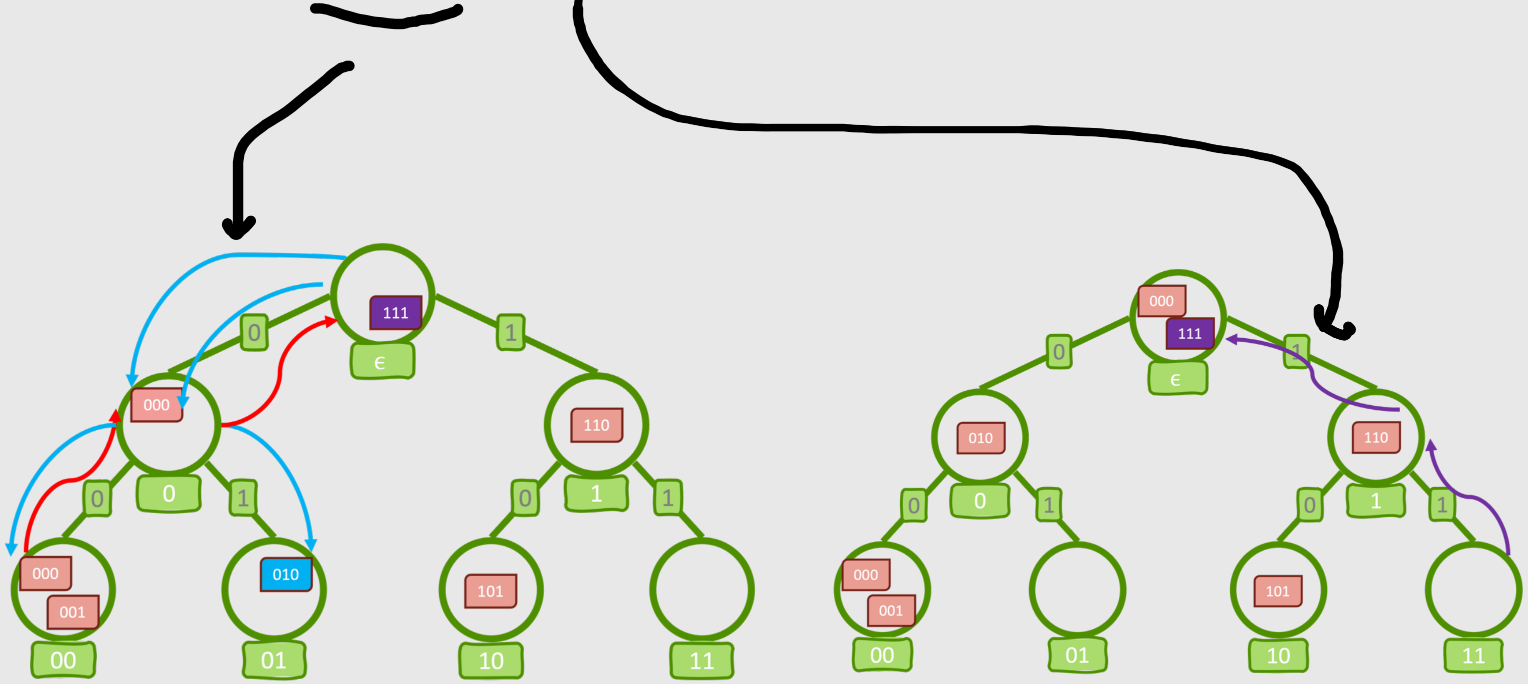
Proof sketch:

- 1 for pull-up
- Fractional occupancy ensures success

  after $\left\lceil \frac{1}{1-f} \right\rceil$ tries, in expectation



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- **Property 2:**

  Access cost of SeedTree $\leq 2 - \log(f) \cdot$Access cost in MRU Tree.

- **Most Recently Used (MRU) Tree:**

  More recently accessed items ⇒ Lower level in the tree

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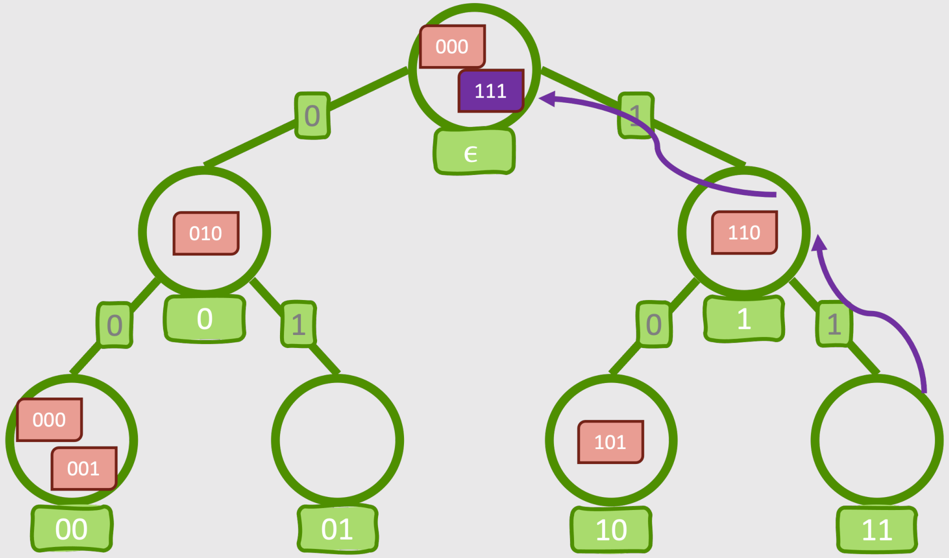
Proof sketch:

- Recent ones go to the root
- Probability of going a level down decreases exponentially per level



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Access cost in MRU Tree $\leq (1 + e)$ ·Access cost of OPT.

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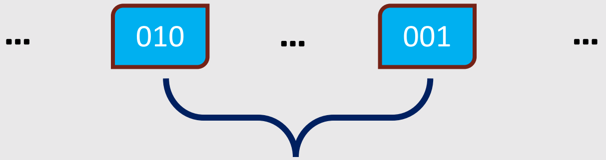
- Property 3:

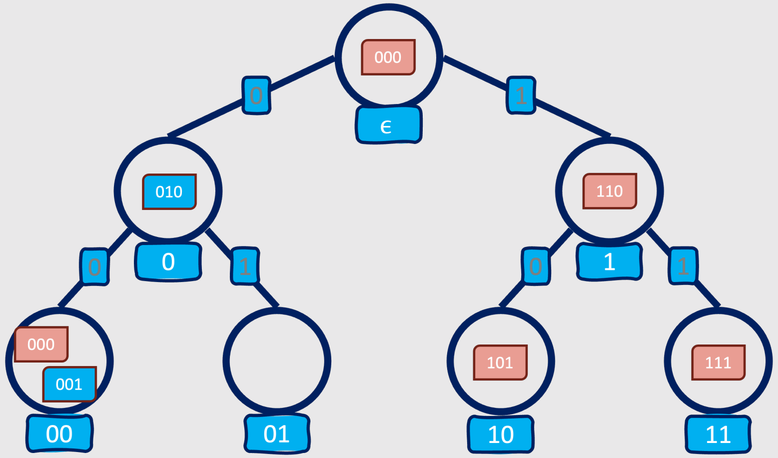
Access cost in MRU Tree $\leq (1 + e) \cdot$ Access cost of OPT.

Proof sketch:

- Potential function analysis based on inversions
- Inversion: item with lower level but accessed earlier

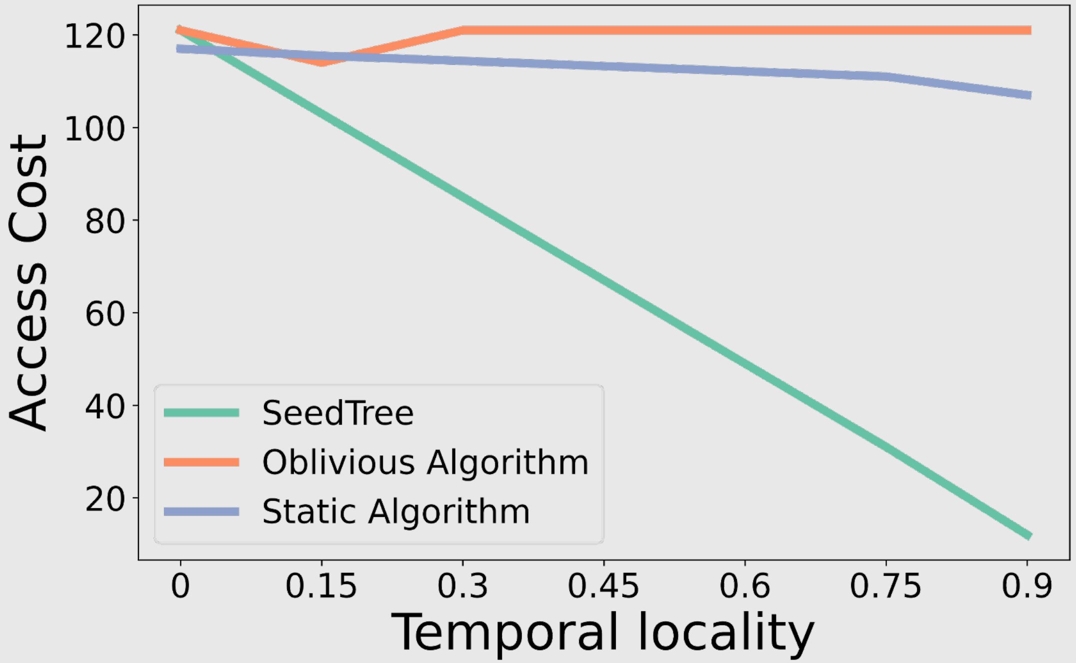


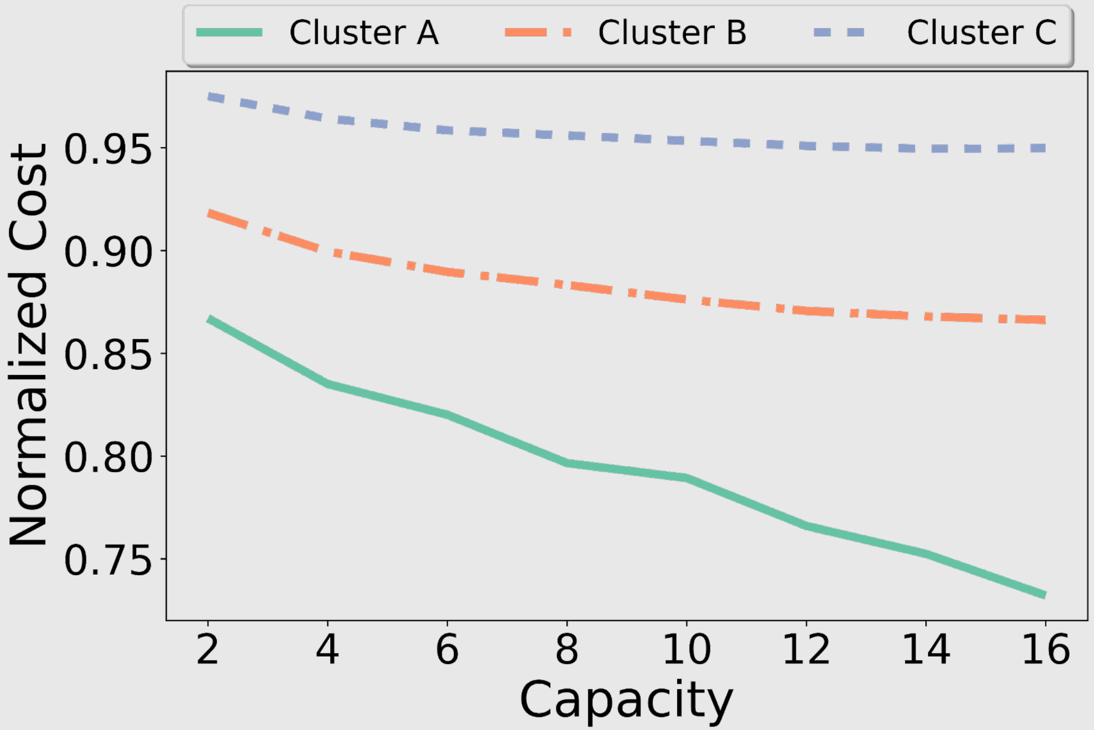
Their last accesses.



OPT's tree

Performance

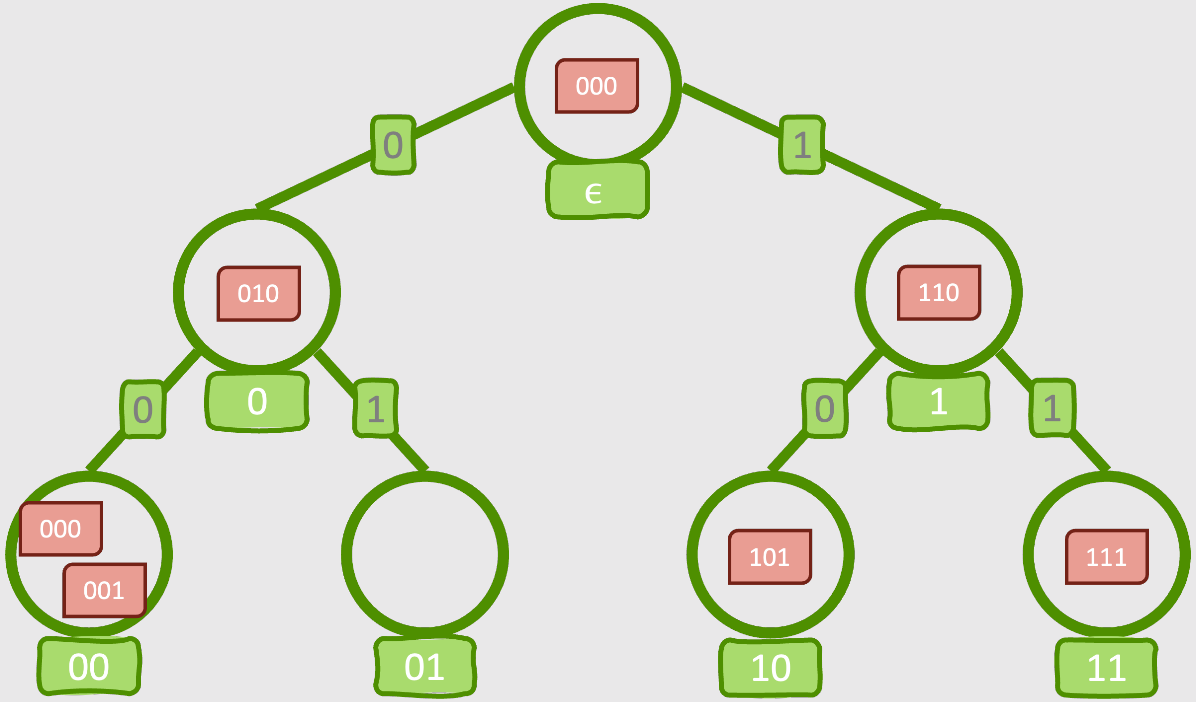




github.com/inet-tub/SeedTree

Conclusion

- Designing a constant competitive algorithm, utilizing randomization in each step.
- Introducing the notion of capacity and item movement for self-adjusting trees.
- Showing significant improvements in the algorithm given inputs with high temporal locality.



Thank You!

Full paper:
arxiv.org/pdf/2301.03074.pdf

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